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Integrating Lot-Sizing Problems Under Uncertainty

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"No que diz respeito ao empenho, ao compromisso, ao esforço, à dedicação, não existe meio termo. Ou você faz uma coisa bem feita ou não faz."

Ayrton Senna

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Abstract

Nowadays, industries' major concerns relate to faster reaction to customer needs and cost reduction in order to increase competitiveness. In this context, lot-sizing problems play a crucial role in production planning to satisfy market demand at the lowest possible cost while meeting the production requirements. However, the increasing complexity of businesses triggered the difficulty in coordinating decisions among different supply chain stages and hierarchical levels. In addition, uncertainty arising from internal and external sources complicates even more the task of providing good feasible solutions for lot-sizing.

In the past years, practitioners and researchers realized that production planning decisions could be more coordinated and costs reduced if more realistic models could incorporate uncertainty sources and lot-sizing could be integrated to other relevant decisions. Nevertheless, one of the major hurdles in optimizing integrated lot-sizing decisions under uncertainty is delivering high quality solutions with low computational effort. Both deterministic models and known solution approaches suffer from lack of effectiveness and efficiency. Consequently, this work aims to efficiently optimize integrated decisions of production lot-sizing with other decisions under various sources of uncertainty. Therefore, two research streams are followed to understand how to better approach specific integrated lot-sizing problems under uncertainty.

The first focuses on the development of integrated lot-sizing mathematical programming models under uncertainty and the comparison and assessment of the main advantages and limitations of each modeling approach. In this stream, stochastic programming and robust optimization models are developed and assessed through a simulation experiment based on Monte Carlo simulation. The models focused on the integration of lot-sizing and scheduling decisions are evaluated for several instances characteristics and settings in terms of average cost, risks and computational runtime. Therefore, this study allows for choosing the most suitable modeling approach according to different circumstances and decision maker preferences.

The second stream is on exact and hybrid solution approaches that help solving large-scale integrated and uncertain models in order to reach high quality solutions in adequate time. In this stream, decomposition methods and acceleration schemes are proposed to efficiently solve a two-stage stochastic programming model that integrates lot-sizing, tactical planning and supplier selection decisions under several sources of uncertainty. In a second moment, approximation and adaptation heuristic strategies are developed to address the integration of lot-sizing and scheduling decisions under multistage demand uncertainty. All the solution methods proposed are compared to standard approaches or evaluated via a simulation experiment in order to assess their solution quality and computational efficiency.

Resumo

Atualmente, as principais preocupações das indústrias estão relacionadas com a necessidade de uma reação mais rápida aos desejos dos consumidores e de uma redução dos custos, afim de aumentar a competitividade. Neste contexto, o dimensionamento de lotes desempenha um papel crucial no planeamento da produção por forma a satisfazer os requisitos de produção e a procura do mercado ao menor custo possível. No entanto, a crescente complexidade das empresas alavancou a dificuldade em coordenar decisões entre diferentes estágios da cadeia de abastecimento e níveis hierárquicos. Além disso, a incerteza que provém de fontes internas e externas complica ainda mais a tarefa de gerar soluções factíveis de boa qualidade para o problema de dimensionamento de lotes.

Nos últimos anos, planeadores e investigadores perceberam que as decisões poderiam ser mais coordenadas e os custos reduzidos se fossem utilizados modelos mais realistas que incorporassem fontes significativas de incerteza e integrassem as decisões de dimensionamento de lote com outras relevantes. No entanto, um dos principais obstáculos em otimizar decisões integradas de dimensionamento de lote sob incerteza é o elevado esforço computacional necessário para obter soluções de alta qualidade. Nesta linha, este trabalho visa otimizar de maneira eficiente as decisões integradas de dimensionamento de lotes de produção com outras decisões sob várias fontes de incerteza. Para tal, duas correntes de investigação são seguidas afim de compreender como melhor abordar problemas integrados de dimensionamento de lotes sob incerteza.

A primeira corrente é centrada no desenvolvimento de modelos de programação matemática que integrem decisões de dimensionamento de lotes sob incerteza, e na comparação e avaliação das principais vantagens e limitações de cada abordagem de modelação. Nesta corrente, modelos de programação estocástica e otimização robusta são desenvolvidos e avaliados através de um experimento computacional baseado na simulação de Monte Carlo. Os modelos, focados na integração das decisões de dimensionamento e sequenciamento de lotes, são avaliados para diferentes características de instâncias e configurações em termos de custo médio, riscos e tempo computacional. Assim, este estudo permite selecionar a abordagem de modelação mais adequada de acordo com diferentes circunstâncias e preferências dos tomadores de decisão.

A segunda corrente foca-se em soluções exatas e híbridas que resolvam os modelos integrados de grande escala e produzam soluções de qualidade elevada em um tempo adequado. Nesta corrente, métodos de decomposição e esquemas de aceleração são propostos para resolver eficientemente um modelo de programação estocástica de dois estágios que integra decisões de dimensionamento de lote, planeamento tático e seleção de fornecedores sob diversas fontes de incerteza. Em um segundo momento, estratégias heurísticas de aproximação e adaptação são desenvolvidas para abordar a integração das decisões de dimensionamento e sequenciamento de lotes sob demanda incerta em vários estágios. Todos os métodos de solução propostos são comparados com abordagens padrão ou avaliados através de um experimento de simulação afim de avaliar a qualidade da solução e eficiência computacional.

Contents

Chapter 1

Motivation and overview

1.1. Problem setting and statement

Most of global supply chains are complex systems with several interdependent suppliers, factories, distribution centers and customers (Sethi et al., 2002). Supply chains are usually subject to a wide variety of uncertainty sources that act at different levels, from raw-materials price to machine setup-time and customers demand. In addition, many decisions in the production stage are taken without considering issues from other supply chain stages. The practice of underestimating uncertainty or not taking joint decisions may jeopardize the companies goals of reducing costs and satisfying customers with high service levels. Within the several departments/processes of a company, the production planning process is key in linking the commercial/marketing efforts with the operational reality.

In particular, decisions of production lot-sizing have an extremely important contribution in defining the quantities and the timings to produce a specific product in a medium-term planning horizon (Karimi et al., 2003). For such decisions, when uncertainty is critical, deterministic models can lead to non-optimal or infeasible solutions. Several works describe the importance of addressing uncertainty in mathematical models for supply chain and planning problems (Mula et al., 2006; Peidro et al., 2009; Aissaoui et al., 2007).

In some settings, neglecting demand uncertainty in lot-sizing models can be considered an inaccuracy (Brandimarte, 2006). Moreover, if a system has limited capacity, uncertainty in processing or setup times may also generate infeasible solutions. Uncertainty in lot-sizing models is presented in several practical applications. For example, Yano and Lee (1995) describe the utilization of random-yields in lot-sizing models in many areas, such as electronic industry and chemical processes.

Uncertainty can be defined as the difference between the amount of information that is available and the quantity of information required to successfully accomplish a task (Galbraith, 1995; Peidro et al., 2009). According to Ho (1989), uncertainty can be categorized in two groups: environmental (exogenous) and system (endogenous). A classical example of the environmental group in industrial problems is demand uncertainty, which can lead to inventory excess or product shortage, increasing the inventory costs or the dissatisfaction of customers, respectively. On the other hand, system uncertainties take place within a production system, such as random processing yields, uncertain setup times or random production times, which can also lead to infeasible production plans.

Two of the most used approaches for modeling uncertainty are robust optimization and stochastic programming models (Gorissen et al., 2015). Both approaches have distinct advantages and drawbacks as it will be discussed later in this dissertation. Still, there is a lack of a systematic methodology that adequately compares the different uncertainty modeling approaches (Sahinidis, 2004). A standard comparison methodology is important in order to assess the performance of the different models concerning the solution quality, efficiency and risk.

Besides the uncertainty issue, in many industrial applications there is a need to consider the integration of lot-sizing with other decisions with the purpose of better coordinating decisions and reducing costs. For instance, in process industries it is necessary to simultaneously take lot-sizing and scheduling decisions to make use of the production capacity as efficiently as possible (Clark et al., 2011).

Decoupled lot-sizing decisions from other supply chain stages (e.g., procurement and distribution) or decision levels (tactical and operational) can lead to sub-optimal solutions. To achieve global optimal solutions, the planning decisions of different levels and/or stages should be taken into account (Maravelias and Sung, 2009).

To integrate different planning decisions into a single model, information between sub-systems needs to be reliable and shared. Moreover, integrated lot-sizing models are commonly larger and more computationally complex to solve than the decoupled models. Usually, more complex planning models should trade-off the level of realism and computational tractability (Clark et al., 2011). To accelerate this integration, it is required a depth understanding of the problem structure in order to build efficient models. This knowledge enables the adoption of adequate mathematical modeling and solution techniques to solve the related problems efficiently and achieve proper solutions. These two elements (uncertainty and integration) should be incorporated in the design of production planning models in order to support real-world decisions. It is possible to mathematically define the integrated lot-sizing problem by means of the following model:

$$\text{minimize} \quad \sum_{j=1}^{|J|}\sum_{t=1}^{|T|}(s_j x_{j,t} + h_j I_{j,t} + h_j^- I_{j,t}^-) + c^T y \tag{1.1}$$

subject to

$$\begin{aligned}
I_{j,t} - I_{j,t}^- = I_{j,t-1} - I_{j,t-1}^- + q_{j,t} - d_{j,t} \quad & \forall\, j \in J, t \in T, & (1.2)\\
p_j q_{j,t} \le C_t x_{j,t} \quad & \forall\, j \in J, t \in T, & (1.3)\\
\sum_{j=1}^{|J|} p_j q_{j,t} \le C_t \quad & \forall\, t \in T, & (1.4)\\
Ax + By = E, \quad & & (1.5)\\
Dy = F, \quad & & (1.6)\\
x_{j,t} \in \{0,1\} \quad & \forall\, j \in J, t \in T, & (1.7)\\
I_{j,t}, I_{j,t}^-, q_{j,t} \ge 0 \quad & \forall\, j \in J, t \in T, & (1.8)\\
y \in \{0,1\} \vee y \ge 0, \quad & & (1.9)
\end{aligned}$$

where J is the set of products and T the set of time periods. $I_{j,t}$ refers to the inventory decision for item j at the period t and $I_{j,t}^-$ the respective backlog decision. $q_{j,t}$ defines the production quantity of item j in period t and $x_{j,t}$ equals one in case a setup for item j occurs in period t, and 0 otherwise. y is the variables vector for the problem to be integrated and x is the vector of the lot-sizing variables $(I_{j,t}, I^-_{j,t}, q_{j,t}, x_{j,t})$. The parameter C_t represents the available production capacity in period t, $d_{j,t}$ the demand for item j in period t, $I_{j,0}$ the product j initial inventory and p_j the required capacity to produce item j. s_j is the setup cost of item j, h^-_j is the shortage cost and h_j is the holding cost of item j. The vector c contains all the costs related to the secondary (integrated) problem. Moreover, A, D and B are the matrices that multiply the vectors y and x. E and F are the right-hand vectors.

Objective function (1.1) minimizes the sum of setup, shortage and holding costs for the planning horizon, which defines the total lot-sizing costs, as well as the costs of the secondary integrated problem. Constraints (1.2) establish the inventory balance. Constraints (1.3) guarantee that item j is only produced if the machine is set up accordingly. Constraints (1.4) ensure that production capacity is respected. Constraints (1.5) couple the lot-sizing problem with the secondary problem. Constraints (1.6) define the space of the secondary problem. Constraints (1.7) - (1.9) are the variables domain-related constraints.

The problem can also have endogenous and/or exogenous uncertainty. Endogenous uncertainty can be incorporated into the model by assuming, for instance, that processing time is uncertain. Therefore, p_j would be transformed into an uncertain parameter: $\hat{p}_j$. If the problem contains exogenous uncertainty sources, such as demand, parameter $d_{j,t}$ would have to be uncertain: $\hat{d}_{j,t}$. Uncertainty can also occur in the secondary problem, such as in its related costs: $\hat{c}^T y$. Besides that, the model that contains these uncertainty parameters should be transformed into its solvable counterpart using some of the uncertainty modeling approaches, such as robust optimization or stochastic programming.

1.2. Research objectives

The main objective of this research is to investigate the integration of lot-sizing decisions with other relevant problems, in different supply chain stages or hierarchical levels, when uncertainty is present. This objective is threefold: 1) understand how to better approach integrated lot-sizing problems under uncertainty with other relevant problems; 2) formulate adequate models for specific integrated lot-sizing problems under uncertainty; 3) develop suitable solution techniques to solve the models in an efficient manner. These goals are interconnected since the last two are fundamental to achieve a clear understanding of the first one.

The scope of this research lies on the development of theoretical models and solution techniques that can improve the lot-sizing decisions within the production planning processes. The research is not focused on any specific industry. Nonetheless, the models to be developed will cover challenges that industries face nowadays, according to the scientific literature and practitioners experience. Moreover, we intend to formulate models that are capable of managing the impact of the critical uncertainty sources in the supply chain.

There are several ways to formulate a problem using uncertainty modeling approaches and specific techniques to improve the solving efficiency. Therefore, the performance of models should be evaluated in terms of solution quality, risk and computational complexity. We aim to develop and use a methodology to perform a systematic comparison and evaluate the different models outcomes and expose their main trade-offs.

The development and usage of solution techniques to solve the models proposed play a crucial role in this research. As stated before, the main difficulty of integrated models under uncertainty lies on its solving efficiency. Therefore, this research aims to develop modeling techniques and solution methods in order to solve the integrated models more efficiently. Decomposition, approximation and relaxation approaches are usually the main solution techniques used to solve mathematical models under uncertainty. They will be revisited for our integrated models, as well as alternative models and valid inequalities that have been derived by several authors. By going one step further, the research questions read:

Research question 1:

What is the most adequate approach to model specific integrated lot-sizing problems under uncertainty?

Lot-sizing problems can be integrated with other problems in order to reach global optimality among the problems considered. However, there are many alternatives to integrate and formulate a lot-sizing problem. Therefore, the model structure should be taken into account in order to bring a more computationally tractable formulation for the problem. To that end, different lot-sizing models (e.g., Capacitated Lot-sizing Problem, Discrete Lot-sizing and Scheduling Problem) and reformulations (e.g., simple plant-location formulation) that have been proposed in the literature should be considered. In order to accelerate the convergence of the models, reformulations, such as convex hull reformulation and valid inequalities, should also be properly applied.

Stochastic programming and robust optimization are the two main approaches used to incorporate uncertainty in optimization models (Gorissen et al., 2015). Nevertheless, usually these two approaches are applied in difference circumstances. Stochastic programming models are generally used to optimize expected values or when recursive decisions are required. On the other hand, robust optimization approaches are used when it is not possible to generate scenarios or to deliver risk averse solutions. Despite the different applications, we intend to establish a fair and systematic evaluation method to compare both modeling approaches. Hence, these approaches will be evaluated in terms of solution quality, computational efficiency and risks measures. Despite all the conducted research, the community does not have a clear roadmap on which modelling approach to use for a given setting.

It is also important to quantify how better the solutions provided by these uncertainty and integrated models are, when compared to the solutions of deterministic and decoupled models. Therefore, we intend to perform a study to compare the main advantages and drawbacks of the developed models, considering the problem characteristics (e.g., associated costs, production capacity assumed, typology of integrated problems, the level and sources of uncertainty) and uncertainty components (e.g., budget of uncertainty, variability level, distribution curve assumed, number of scenarios or risk measures incorporated) of each modeling approach.

In particular we focus on (i) the integration of production lot-sizing with procurement and distribution planning, supplier selection and product branding decisions under demand, price, lead-time and raw-material availability uncertainty; (ii) and on the integration of lot-sizing with scheduling decisions under (multistage) demand uncertainty.

Research question 2:
What are the best strategies to efficiently solve specific integrated lot-sizing problems under uncertainty?

There are several known solution techniques that can be used to solve optimization problems, from exact methods to metaheuristics and hybrid procedures. Since some static robust optimization approaches generally maintain the computational tractability of the deterministic models, standard solution techniques are often suitable to these problems to optimality. However, stochastic programming and adjustable robust optimization models usually require more computational effort whenever there are a high number of scenarios or integer variables, respectively. Therefore, these modelling approaches require specific solution techniques to improve the solving efficiency, such as splitting sets methods (Bertsimas and Caramanis, 2010; Postek and Den Hertog), decomposition techniques (e.g., L-shaped, progressive hedging, Lagrangian decomposition and cross decomposition) and heuristic approaches (e.g., rolling-horizon schemes and approximation methods).

As mentioned before, the formulation of the models also affects the solving efficiency and solution quality. Therefore, different lot-sizing and uncertainty models, reformulations techniques and valid inequalities should also be considered to address the intractability of the problems.

Finally, it is fundamental to compare several solution techniques and modeling approaches in terms of solution quality and computational runtime in order to define the most suitable strategy for solving the specific problems focused. This comparison study is essential to answer this research question and also to fill the gap of studies that compare the performance of modeling and solution techniques for uncertainty problems.

In particular we focus on (i) the integration of production lot-sizing with procurement and distribution planning, supplier selection and product branding decisions under demand, price, lead-time and raw-material availability uncertainty; (ii) and on the integration of lot-sizing with scheduling decisions under (multistage) demand uncertainty.

1.3. Thesis synopsis

This thesis is organized in the following way. In this introductory chapter, the first section characterizes the problem and illustrates its practical relevance. Then, the second section describes the research objectives and raises the main research questions that this research aims to answer. This section presents the structure of the thesis and describes the content of each chapter.

The second chapter reviews the main scientific literature and is divided into four sections. In the first section we describe the main deterministic lot-sizing models and their variants, while in the second and third sections we review the main contributions on lot-sizing under uncertainty and on integrating lot-sizing problems, respectively. In the fourth section, we report the main literature highlights and the gaps identified.

Chapters 3, 4 and 5 of this thesis consist in submitted papers to international journals. Each chapter (paper) helps answering one or the two research questions raised. Chapter 3 addresses the first and second research questions. It proposes a two-stage stochastic programming model and a decomposition approach for solving an integrated strategic and tactical planning problem for the processing food industry under four sources of uncertainty. Chapter 4 discusses the main trade-offs of uncertainty modelling approaches for the integrated lot-sizing and scheduling problem under demand uncertainty, which is mainly aligned to the objectives of the research question 1. Chapter 5 proposes heuristic strategies to efficiently solve the integrated lot-sizing and scheduling problem under multistage demand uncertainty. Moreover, it compares several models and strategies using a simulation scheme to help answering the first and second research question.

In Chapter 3, we develop a two-stage stochastic programming model to address integrated decisions of supplier selection and procurement, lot-sizing and distribution planning under four sources of uncertainty: lead-time, raw material availability, price of raw material and final product demand. The model is focused on the food processing industry, incorporating its main features, such as raw material and product perishability and also consumer willingness to pay according to product shelf-life. To solve the problem in an efficient manner, we reformulate the model using the convex hull reformation, propose two versions of Benders decomposition and apply acceleration techniques. The summarized contributions of this work are two-fold: firstly, the development of a new model that considers the main features of the food processing industry, incorporates several sources of uncertainty and integrates tactical planning and strategic sourcing decisions. Secondly, the proposal of decomposition and acceleration methods to solve the two-stage stochastic programming model more efficiently. In this work, the PhD candidate mainly contributed by validating the models, developing the decomposition and acceleration methods, performing the computational study and analyzing the results.

Chapter 4 proposes a robust optimization model based on polyhedral uncertainty sets and a two-stage stochastic programming model to address demand uncertainty in the lot-sizing and scheduling problem. In this problem, production and setup variables are taken and fixed for the whole planning horizon. The main objective is to analyze the main advantages and drawbacks of the robust optimization and stochastic programming in the lot-sizing and scheduling context. To that end, we propose a Monte Carlo simulation to evaluate the trade-offs of the uncertainty modeling approaches in terms of average cost, risk and computational complexity. Based on the simulation outcomes, we provide two flowcharts to assist decisions makers to best select the right modeling approaches and uncertainty parameters according to different instance characteristics and decision maker preferences. The main contributions of this work are the development of a robust optimization for the problem, a methodology to systematically evaluate the models that incorporate uncertainty, and the proposal of guidelines to help decision makers to effectively solve the problem according to different preferences and problem characteristics. The contributions of the PhD candidate in this work are the implementation and validation of the models, the proposal of the Monte Carlo simulation method to evaluate the uncertainty approaches, the development of the computational experiment and the analysis of the simulation outcomes.

In the fifth chapter, two heuristic strategies are proposed to efficiently address the general lot-sizing and scheduling problem under multistage demand uncertainty. In this problem, production, scheduling and inventory decisions can be adjusted in every time period. The standard approaches to deal with the problem are the multistage stochastic programming or the deterministic model embedded with rolling-horizon planning schemes. The strategies proposed are combinations of uncertainty modelling approaches and rolling-horizon planing schemes. The first strategy adapts the two-stage stochastic programming and robust optimization models to the multistage setting using a shrinking-horizon planning scheme. The second combines an approximate multistage stochastic programming model and an approximate adjustable robust optimization model with the rolling-horizon planning scheme in order to make them more tractable. To compare the strategies and the models proposed we develop a simulation experiment based on Monte Carlo simulation and rolling-horizon scheme.

This work has two main contributions, the first is the development of models and efficient strategies to tackle multistage demand uncertainty in the general lot-sizing and scheduling problem. To the best of our knowledge, it is the first time that adjustable robust optimization is applied in lot-sizing and scheduling problems. Also, it seems that approximation strategies have never been used before to address uncertainty problems. The second contribution is the proposal of a simulation experiment to evaluate and compare the models and strategies developed in a multistage demand uncertainty setting. In this work, the contributions of the PhD candidate are the proposal and development of the models, solution strategies and computational experiments.

Finally, the last chapter summarizes the work contributions and based on them, provides answers to the research questions raised. This chapter also presents new research directions.

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Chapter 2

Literature review

This chapter is divided in four sections. In the first section deterministic lot-sizing models are introduced. Next, lot-sizing models under uncertainty are analysed. In the third section, a literature review on integrated lot-sizing problems in both deterministic and uncertainty settings is performed. In the literature review of integrated lot-sizing under uncertainty, the papers are classified by the problem characteristics, mathematical modeling approach and solution techniques used. The main objective is to understand the state-of-the-art regarding models for integrated lot-sizing under uncertainty and identify possible gaps that can be investigated in the subsequent steps of this research. The chapter concludes in the forth section with a critical view over the literature in this field.

2.1. Deterministic lot-sizing problems

This literature review details the main capacitated lot-sizing models and their variants. It is based on the main literature reviews of capacitated lot-sizing problem (Quadt and Kuhn, 2008; Karimi et al., 2003; Gicquel et al., 2009) and lot-sizing and scheduling problems (Drexl and Kimms, 1997; Copil et al., 2016). We believe that this literature review is important to frame the major progresses in deterministic lot-sizing problems.

The lot-sizing models can be distinguished into two major categories of time discretization. The first is the big bucket models, in which planning horizons are typically considered to be less than 6 months (Drexl and Kimms, 1997) and several items are allowed to be produced within a time period. In the small bucket models category, each time period is discretized in smaller periods (micro-period), and only one product can be produced within a micro-period. This assumption allows for lot-sizing and scheduling decisions to be taken jointly in a straightforward manner (Gicquel et al., 2009).

2.1.1 Big bucket models

The capacitated lot-sizing problem (CLSP) differs from the classical economical order quantity (EOQ), economic lot scheduling (ELSP) and Wagner-Whitin (WW) problems. In the classical EOQ, the demand is stationary and the capacity unlimited. In the ELSP, there is capacity limit, but the demand is stationary. In the WW problem, the demand is dynamic, but there is no capacity limit. In the CLSP, the capacity is limited and the demand is dynamic.

The CLSP was proven to be NP-hard (Florian et al., 1980; Bitran and Yanasse, 1982). Consequently, when it is integrated with other problems, such as scheduling, the complexity to solve it increases substantially - for instance, the general lot-sizing and scheduling problem (GLSP) is also known to be NP-hard (Jodlbauer, 2006).

As in other problems, the way the CLSP is modeled impacts directly on its linear programming relaxation and on the performance of the branch-and-bound algorithm. Previous research, such as Stadtler (2003) and Brandimarte (2006) reformulate the CLSP as a simple plant-location model, attempting to provide stronger relaxations and consequently better lower bounds for the model.

We describe below a standard formulation of the CLSP as a mixed-integer programming model (Drexl and Kimms, 1997; Quadt and Kuhn, 2008; Gicquel et al., 2009):

$$\text{minimize} \quad \sum_{j=1}^{|J|}\sum_{t=1}^{|T|}(s_j x_{j,t} + h_j I_{j,t}) \tag{2.1}$$

subject to

$$I_{j,t} = I_{j,t-1} + q_{j,t} - d_{j,t} \quad \forall\, j \in J, t \in T, \tag{2.2}$$

$$p_j q_{j,t} \leq C_t x_{j,t} \quad \forall\, j \in J, t \in T, \tag{2.3}$$

$$\sum_{j=1}^{|J|} p_j q_{j,t} \leq C_t \quad \forall\, t \in T, \tag{2.4}$$

$$x_{j,t} \in \{0,1\} \quad \forall\, j \in J, t \in T, \tag{2.5}$$

$$I_{j,t}, q_{j,t} \geq 0 \quad \forall\, j \in J, t \in T, \tag{2.6}$$

where J is the set of products and T the set of time periods. $|.|$ is the cardinality of the set $\{.\}$. Decision variables $I_{j,t}$ refer to the inventory of item j in period t, $q_{j,t}$ to the production quantity of item j in period t and $x_{j,t}$ equals one in case a setup of item j occurs in period t, and 0 otherwise. The parameter C_t represents the available production capacity in period t, $d_{j,t}$ the demand of item j in period t, $I_{j,0}$ product j initial inventory and p_j is the required capacity to produce item j. The setup cost of item j is given by s_j and h_j is the holding cost of item j.

The objective function (2.1) minimizes the sum of setup and holding costs for the planning period. Constraints (2.2) establish the inventory balance. Constraints (2.3) guarantee that item j will only be produced if the machine is set up for the respective item. Constraints (2.4) ensure that production capacity is respected. Constraints (2.5) set the variables x_{jt} as binaries and constraints (2.6) set variables $I_{j,t}$ and $q_{j,t}$ as non-negative.

2.1.2 Small bucket models

The two most known small bucket lot-sizing models are the Continuous Setup Lot-sizing Problem (CSLP) and the Discrete Lot-sizing and Scheduling Problem (DLSP). In the CSLP, it is assumed that the planning is made within micro-periods, in which at most one product can be produced in each time (micro-)period. In the CSLP changeover variables are required to indicate that a new item is produced in the beginning of a period. The mixed-integer programming model of the CSLP is described below:

$$\text{minimize} \quad \sum_{j=1}^{|J|} \sum_{t=1}^{|T|} (s_j x_{j,t} + h_j I_{j,t}) \tag{2.7}$$

subject to

$$I_{j,t} = I_{j,t-1} + q_{j,t} - d_{j,t} \quad \forall\, j \in J, t \in T, \tag{2.8}$$

$$p_j q_{j,t} \leq C_t y_{j,t} \quad \forall\, j \in J, t \in T, \tag{2.9}$$

$$\sum_{j=1}^{|J|} y_{j,t} \leq 1 \quad \forall\, t \in T, \tag{2.10}$$

$$x_{j,t} \geq y_{j,t} - y_{j,t-1} \quad \forall\, j \in J, t \in T, \tag{2.11}$$

$$y_{j,t} \in \{0,1\} \quad \forall\, j \in J, t \in T, \tag{2.12}$$

$$I_{j,t}, q_{j,t}, x_{j,t} \geq 0 \quad \forall\, j \in J, t \in T. \tag{2.13}$$

The sets, parameters and most of the variables are equal to the CLSP model. The exception comes from the variables $y_{j,t} = 1$ that represent a new setup of item j occurring in period t, and $x_{j,t}$ that equals one when there is a changeover to product j in period t.

In the CSLP model there are three different constraints from the CLSP model. The constraints (2.10) ensure that only one setup occurs in a period. Constraints (2.11) ensure that the setup costs only occur when there is a setup for a new product lot and the constraints (2.13) set the variables $x_{j,t}$ as non-negative.

A drawback of the CSLP is that if in a given period a product is not produced using the full machine capacity, then production capacity will remain unused. To workaround this issue, it is suggested a model where the remaining production capacity may be used to produce a second item. The proportional lot-sizing and scheduling problem (PLSP) allows for the scheduling of two items in a single period. To obtain the PLSP model the constraints (2.9) are replaced by:

$$p_j q_{j,t} \leq C_t (y_{j,t} + y_{j,t-1}) \quad \forall\, j \in J, t \in T, \tag{2.14}$$

and the constraints that ensure that the production capacity is respected are added:

$$\sum_{j=1}^{|J|} p_j q_{j,t} \leq C_t \quad \forall\, t \in T. \tag{2.15}$$

The DLSP is another well-known small bucket model. The DLSP was also proven to be NP-hard (Drexl and Kimms, 1997) and the difference from the CSLP is that if a product is chosen to be produced, it must use the full production capacity. In other words, the constraints (2.9) are replaced by:

$$p_j q_{j,t} = C_t y_{j,t} \quad \forall\ j \in J, t \in T. \tag{2.16}$$

2.1.3 Lot-sizing variants

Besides the different aforementioned time period discretizations, the lot-sizing problem can have several variants in terms of capacity, number of items, number of machines, inclusion of back-orders and others. Let us characterize the main variants of lot-sizing problems and the main approaches used to solve them based on the previous existent literature reviews (Karimi et al., 2003; Quadt and Kuhn, 2008; Gicquel et al., 2009).

2.1.3.1 Capacity

Resources such as manpower, machines and equipments can limit the capacity of the production system. When capacity is not taken into account the problem is known as uncapacitated lot-sizing problem, whereas if capacity is taken into account the problem is considered capacitated. Capacitated resources directly increase the complexity of the lot-sizing problems: note that it is possible to solve uncapacitated lot-sizing in polynomial time, while the CSLP is NP-hard for both single and multi-product settings.

To modify the CLSP model into the uncapacitated version, constraints (2.4) should be removed and the parameter C_t must be replaced in constraints (2.3) by a Big-M parameter, for instance:

$$p_j q_{j,t} \leq \sum_{t'=t}^{|T|} d_{j,t'} x_{j,t} \quad \forall\ j \in J, t \in T. \tag{2.17}$$

It is important to note that without capacity requirements, it is possible to disaggregate the multi-product model into $|J|$ single-product models.

2.1.3.2 Number of products

Number of final products is another characteristic that directly impacts on the problem complexity. Naturally, the complexity of the multi-item lot-sizing is higher than the single item lot-sizing. Single item lot-sizing problem is a well studied problem. Brahimi et al. (2006) make a detailed literature review of extensions, models and solution techniques of single item lot-sizing problems. To alter the number of products, it is necessary to change the set of products J.

2.1.3.3 Number of machines

Lot-sizing problems with parallel machines present a more realistic setting, with a higher practical relevance. In these cases items have to be produced and assigned to the machines and it is necessary to decide the number of machines to be used in parallel, which makes the production planning more detailed and realistic. This variant is more complex to solve than the single machine version.

The following mathematical model represents the CLSP with parallel unrelated machines (Toledo and Armentano, 2006):

$$\text{minimize} \quad \sum_{j=1}^{|J|}\sum_{t=1}^{|T|}\sum_{m=1}^{|M|}(s_{m,j}x_{m,j,t}+c_{m,j}q_{m,j,t})+\sum_{j=1}^{|J|}\sum_{t=1}^{|T|}h_jI_{j,t} \tag{2.18}$$

subject to

$$I_{j,t}=I_{j,t-1}+\sum_{m=1}^{|M|}q_{m,j,t}-d_{j,t} \qquad \forall\ j\in J,t\in T, \tag{2.19}$$

$$p_jq_{m,j,t}\leq C_{m,t}x_{m,j,t} \qquad \forall\ j\in J,t\in T,m\in M, \tag{2.20}$$

$$\sum_{j=1}^{|J|}(p_{m,j}q_{m,j,t}+f_{m,j}x_{m,j,t})\leq C_{m,t} \qquad \forall\ t\in T,m\in M, \tag{2.21}$$

$$x_{m,j,t}\in\{0,1\} \qquad \forall\ j\in J,t\in T,m\in M, \tag{2.22}$$

$$I_{j,t},q_{m,j,t}\geq 0 \qquad \forall\ j\in J,t\in T,m\in M, \tag{2.23}$$

where M is the set of machines, $c_{m,j}$ is the cost of producing product j on machine i, $p_{m,j}$ is the processing time of product j on machine m, $s_{m,j}$ is the setup cost of product j on machine m and $f_{m,j}$ is the setup time of product j on machine m. The variables $q_{m,j,t}$ and $x_{m,j,t}$ now have the index m that represents the machine where the production or the setup occurs, respectively. The model is similar to the regular CLSP model, being the main difference the incorporation the setup time into the capacity constraints (2.21) and the production costs in the objective function.

2.1.3.4 Back-orders

The possibility to attend a demand after its due period is called back-order or backlogging. When back-orders are incorporated into the models usually a shortage cost is penalized in the objective function. In the lost sales case, demand that is not fulfilled in its due period will not be satisfied at all. The incorporation of back-orders or lost sales may be fundamental in order to reach a feasible plan, because in many real-world cases it is not possible to attend all the demand required for a given time period.

To incorporate back-orders into the big bucket lot-sizing model, the constraints (2.2) should be replaced by:

$$I_{j,t}-I^-_{j,t}=-I^-_{j,t-1}+I_{j,t-1}+q_{j,t}-d_{j,t} \qquad \forall\ j\in J,t\in T, \tag{2.24}$$

where $I^-_{j,t}$ refers to the back-order quantity variable. Moreover, the cost of back-order should be added into the objective function as follows:

$$\text{minimize} \quad \sum_{j=1}^{|J|} \sum_{t=1}^{|T|} (s_j x_{j,t} + h_j I_{j,t} + h_j^- I_{j,t}^-), \tag{2.25}$$

where h_j^- denotes the cost of backlogging item j.

2.1.3.5 Setup structure

Setup costs and times can be divided into two types: sequence independent and dependent. In the former case, the setup decision of the preceding period does not influence the setup time and costs of the subsequent period. In the latter, the opposite happens and therefore, for different production sequences different setup times/costs are incurred. In both cases there exists setup carry-over, if when the product produced in the previous period is produced in the current period, no additional setup is necessary. Another case is family setup, in which the setup time and cost are dependent of manufacturing similarities among the products. It is clear that sequence dependent setups are computationally more complex than the sequence independent.

The following model is an extension of the DLSP model, with sequence dependent setups (Haase, 1996):

$$\text{minimize} \quad \sum_{i=1}^{|J|} \sum_{j=1}^{|J|} \sum_{t=1}^{|T|} s_{i,j} x_{i,j,t} + \sum_{j=1}^{|J|} \sum_{t=1}^{|T|} h_j I_{j,t} \tag{2.26}$$

subject to

$$I_{j,t} = I_{j,t-1} + q_{j,t} - d_{j,t} \quad \forall\, j \in J, t \in T, \tag{2.27}$$

$$p_j q_{j,t} = C_t y_{j,t} \quad \forall\, j \in J, t \in T, \tag{2.28}$$

$$\sum_{j=1}^{|J|} y_{j,t} = 1 \quad \forall\, t \in T, \tag{2.29}$$

$$x_{i,j,t} \geq y_{j,t} + y_{i,t-1} - 1 \quad \forall\, j \in J, t \in T, i \in J, \tag{2.30}$$

$$y_{j,t} \in \{0,1\} \quad \forall\, j \in J, t \in T, \tag{2.31}$$

$$I_{j,t}, q_{j,t} \geq 0 \quad \forall\, j \in J, t \in T, \tag{2.32}$$

$$x_{i,j,t} \geq 0 \quad \forall\, j \in J, t \in T, i \in J, \tag{2.33}$$

where $s_{i,j}$ is the cost of changing the setup from item i to item j and $x_{i,j,t}$ represents the changeover from item i to item j. The remainder variables and parameters are similar to the DLSP model described before. The constraints (2.29) force a setup to occur in every period (which includes setup for the idle item "0"). Constraints (2.30) force a setup cost to occur every time that setup changes from item i to item j. If the triangle inequality ($s_{i,j} \leq s_{i,k} + s_{k,j}$ $\forall\, i, j, k \in J$) does not hold, production of unnecessary items may occur, and the following constraints can be added to avoid such situation:

$$I_{j,|T|} < \frac{C_t}{p_j} \quad \forall\ j \in J. \tag{2.34}$$

2.1.3.6 Demand

Demand can be static, dynamic or random. A static demand means that the demand is equal for any period considered. In the dynamic variant, the demand can vary from period to period, but it is known in advance. Random demand is not known in advance and its value is uncertain. Moreover, demand can be classified as dependent and independent. The independent demand refers to the external demand or customer demand for final products. The dependent demand (also called internal demand) of a product is defined by the final products demand and the requirements of intermediate products to produce final products.

2.1.3.7 Planning horizon

The planning horizon can be distinguished in finite or infinite. Infinite planning horizon is often used with a stationary demand assumption and addressed in EOQ and ELSP. Finite planning horizon is usually associated with dynamic demand.

2.1.3.8 Number of levels

There are two main types of production system considered in lot-sizing problems: single-level and multi-level. In single-level lot-sizing only independent demand products are considered and intermediate products are not accounted for producing final products. In multi-level lot-sizing problems, intermediate items have to be produced in order to be used to produce or assemble final products. In other words, there exists dependent demand for intermediate items. Multi-level lot-sizing problems are also more complex to solve than the single-level problems. Kimms (2012) makes an extensive research on multi-level lot-sizing and scheduling problem and extensions to it.

Basically, it is possible to incorporate the multi-level structure into the CLSP model by replacing the constraints (2.9) for:

$$I_{j,t} = I_{j,t-1} + q_{j,t} - d_{j,t} - \sum_{i \in S_j} a_{j,i} q_{j,i} \quad \forall\ j \in J, t \in T, \tag{2.35}$$

and adding the constraints:

$$I_{j,t} \geq \sum_{i \in S_j} \sum_{\tau=t+1}^{min\{t+v_j,|T|\}} a_{j,i} q_{j,\tau} \quad \forall\ j \in J, t = 0, \ldots, |T|-1, \tag{2.36}$$

where S_j is the set of immediate successors of item j, v_j is the lead time of item j and $a_{j,i}$ is the "gozinto" factor, i.e., the quantity of item j needed to produce one item i. The constraints (2.36) add the lead time to attend internal demand.

2.2. Lot-sizing under uncertainty

Several works address uncertainty in their lot-sizing models with the aim of bringing a more realistic perspective to this problem. Usually, uncertainty sources in lot-sizing problems occur in demand or within the production system, for example in production times, yields, lead times and costs related to production and setups.

This literature review focuses on the main works that cover demand uncertainty and system uncertainty (production times, yields and costs). Moreover, we concentrate this literature review on researches that use mathematical models (mostly robust optmization and stochastic programing modelling approaches) to tackle uncertainty in capacitated lot-sizing models for finite planning horizons with dynamic random demand. Most of the publications reviewed are in the main literature reviews of uncertainty in lot-sizing (Yano and Lee, 1995; Sox et al., 1999; Dolgui and Prodhon, 2007; Aloulou et al., 2014).

Several literature review papers address the uncertainty in lot-sizing by many aspects, nonetheless many of them focus on works that aim at obtaining optimal cyclic policies (Yano and Lee, 1995; Dolgui and Prodhon, 2007) and make assumptions of stationary or infinite planning horizon, such as the ELSP (Sox et al., 1999) or focus on different solution techniques (e.g., Markov chain models, simulation, EOQ models, game-theory models) (Yano and Lee, 1995; Dolgui and Prodhon, 2007). Aloulou et al. (2014) are the only authors that provide a detailed taxonomy of the aspects of the problem considered and the solution technique used.

Most authors consider that there are still gaps to be explored in lot-sizing under uncertainty. According to Aloulou et al. (2014), only recent research started to study complex system such as multi-product, multi-period and multi-machine settings. Dolgui and Prodhon (2007) state that safety stocks are common to avoid the risk of shortage, but in some cases it can be expensive, so new approaches to efficiently satisfy the demand and reduce the costs are required. Dolgui and Prodhon (2007) also highlight that few works performed incorporate both lead-time and demand uncertainty in models and it may have high practical value. It is clear that the need for complex models brings a collateral challenge of developing efficient solution approaches, which was identified by Sox et al. (1999).

2.2.1 Demand uncertainty

There is a considerable number of papers addressing demand uncertainty in lot-sizing problems. Most of the works uses multi-stage stochastic programming and heuristics to solve the models. Recently, robust optimization has been used as an alternative method to stochastic programming. Below we describe the main works that contributed to this stream.

Martel et al. (1995) were one of the first authors to develop a two-stage stochastic model for the multiple-item procurement under stochastic demand setting. The model is focused on consumer goods and retail industries. Moreover, the authors use a branch-and-bound algorithm to solve the proposed model.

To reach a more realistic perspective, a considerable number of authors tackle demand uncertainty with multi-stage stochastic models. Brandimarte (2006) develops a multi-stage stochastic model for the stochastic version of the multi-item capacitated lot-sizing problem. The model is formulated as a plant-location-based model, which provides a better linear relaxation and consequently a better performance of the branch-and-bound algorithm. Demand uncertainty is incorporated in the multi-stage stochastic model and a heuristic is proposed to solve it. Sodhi (2005) also formulates a multi-stage stochastic model in order to tackle demand uncertainty for an electronics company. The author also provides two risk measures to manage the demand uncertainty. In order to solve the model standard solvers are used since the variables are all linear and the main ones are scenario independent. Nonetheless, the author suggests techniques to improve the solving efficiency, such as Bender's decomposition, progressive hedging algorithm (PHA) and scenario aggregation approaches.

Tsang et al. (2007) develop a multi-stage stochastic programming model to optimize the capacity planning and investment strategy for the vaccine industry. In addition, the model takes into account uncertainty in demand and incorporates risk measures to be used in the model.

Some researches apply approaches to reduce the computational complexity of the stochastic models. Taskin and Jr. (2010) develop a multi-stage stochastic programming model embedded with scenario reduction techniques to reduce the computational challenges of the model. The underlying problem is the inventory control with stochastic demand. Sodhi and Tang (2009) develop a multi-stage stochastic linear programming to model a supply-chain planning problem with asset–liability management aspects under demand uncertainty. The authors propose sampling scenarios, decomposition techniques and scenario aggregation approaches in order to reduce the stochastic model complexity.

Specific heuristics are also used to solve uncertain lot-sizing problems. Tempelmeier (2011) develops a column generation heuristic in order to solve the capacitated lot-sizing with random demand. Moreover, service level constraint and back-orders are considered. The proposed heuristic is based on column generation with a previous ABC_β heuristic from the same author. Haugen et al. (2001) develop a metaheuristic based on the PHA to solve a single-item stochastic lot-sizing problem. In the problem the demand is uncertain and there is no production capacity. Moreover, the problem is modeled as a multi-stage stochastic program, in which the authors prove optimality for the 3-stage case.

More recently, robust optimization models have become an alternative to tackle uncertainty. Zhang (2011) tackles demand uncertainty for a single product in the uncapacitated lot-sizing problem using robust optimization models. Vairaktarakis (2000) also develops a robust model for the multi-item newsboy with uncertain demand. Ben-Tal et al. (2005) apply robust optimization to lot-sizing of a single-product in a two-echelon supply chain focusing more on supplier and retailer coordination with random demand. Bertsimas and Thiele (2006) propose an interval-based robust optimization to incorporate demand uncertainty in the supply chain control problem.

In cases in which infinite horizon planning or static demand are assumed, other solution approaches can be used, rather than mathematical programming models. For instance, Minner and Silver (2005) and Minner and Silver (2007) rely on Markov-chain based models to tackle random demand for multiple products and establish optimal replenishment policies. In Aloulou et al. (2014), different modeling approaches are used to tackle non-deterministic lot-sizing problems, such as simulation, fuzzy programming and game theory.

Dynamic stochastic programming and specific approaches have also been used. Raa and Aghezzaf (2005) present the single-item lot-sizing problem with demand uncertainty. To tackle the problem, the authors suggest an alternative dynamic probabilistic approach for the multi-stage stochastic programming. Moreover, computational experiments are run to compare the dynamic probabilistic approach, the deterministic model and the static probabilistic approach. The experiment showed that the results of the static probabilistic and dynamic probabilistic approaches are, on average, 1% and 5% better than the deterministic approach, respectively. In addition, the authors develop two-stage and multi-stage stochastic models, but it was not possible to run computational experiments for this ones due to the computational intractability of the models.

Finally, there are stochastic models to compare different production strategies. Leung and Ng (2007) address the production planning for perishable products. The authors formulate a two-stage stochastic programming model to incorporate production postponement strategies and tackle the uncertainty in demand and costs. Computational results compare the postponement strategy with non-postponement strategy and show the relevance of the model to deal with uncertainty.

2.2.1.1 Conceptual models

Demand uncertainty is one of the major sources of uncertainty (Aloulou et al., 2014). In this subsection, we present two stochastic models and one robust model incorporating demand uncertainty.

The two-stage stochastic programming model below considers that the production quantity has to be defined before the demand realization, and the decisions of demand fulfillment and inventory levels have to be taken in the second stage:

$$\text{minimize} \quad \sum_{j=1}^{|J|}\sum_{t=1}^{|T|} s_j x_{j,t} + \sum_{j=1}^{|J|}\sum_{t=1}^{|T|}\sum_{k=1}^{|K|} \pi_k (h_j I_{j,t,k} + h_j^- I_{j,t,k}^-) \tag{2.37}$$

subject to

$$I_{j,t,k} - I_{j,t,k}^- = I_{j,t-1,k} - I_{j,t-1,k}^- + q_{j,t} - d_{j,t,k} \quad \forall\, j \in J, t \in T, k \in K, \tag{2.38}$$

$$p_j q_{j,t} \le C_t x_{j,t} \quad \forall\, j \in J, t \in T, \tag{2.39}$$

$$\sum_{j=1}^{|J|} p_j q_{j,t} \le C_t \quad \forall\, t \in T, \tag{2.40}$$

$$x_{j,t} \in \{0,1\} \quad \forall\, j \in J, t \in T, \tag{2.41}$$

$$I_{j,t,k}, I_{j,t,k}^-, q_{j,t} \ge 0 \quad \forall\, j \in J, t \in T, k \in K, \tag{2.42}$$

where K is the set of scenarios, π_k the probability of occurrence the scenario k, such that

$\sum_{k=1}^{|K|} \pi_k = 1$ and $d_{j,t,k}$ the demand of product j at time t in scenario k. The parameter h_j^- is the shortage cost of product j, the decision variable that controls the stock $I_{j,t,k}$ becomes a second stage variable and $I_{j,t,k}^-$ is added, which denotes the backlog variable.

A considerable number of authors incorporates demand uncertainty using multi-stage stochastic models. Multi-stage stochastic programming is even more intractable than the two stage programming approach (Dyer and Stougie, 2006). However, it can be more adequate to model some specific problems, for instance when it is possible to adjust the production lot size every period. We describe the deterministic equivalent of a multi-stage stochastic model, in which demand is uncertain and the production quantity can be adjusted in every time period (Kazemi Zanjani et al., 2010):

$$\text{minimize} \quad \sum_{j=1}^{|J|}\sum_{t=1}^{|T|}\sum_{k=1}^{|K|} \pi_k (s_j x_{j,t_k,k} + h_j I_{j,t,k} + h_j^- I_{j,t,k}^-) \tag{2.43}$$

subject to

$$I_{j,t_k,k} - I_{j,t_k,k}^- = I_{j,t_{k'},k'} - I_{j,t_{k'},k'}^- + q_{j,t,k} - d_{j,t,k} \quad \forall\ j \in J, k \in K, \tag{2.44}$$

$$p_j q_{j,t_k,k} \leq C_{t_k} x_{j,t_k,k} \quad \forall\ j \in J, k \in K, \tag{2.45}$$

$$\sum_{j=1}^{|J|} p_j q_{j,t_k,k} \leq C_{t_k} \quad \forall\ k \in K, \tag{2.46}$$

$$x_{j,t_k,k} \in \{0,1\} \quad \forall\ j \in J, k \in K, \tag{2.47}$$

$$I_{j,t_k,k}, I_{j,t_k,k}^-, q_{j,t_k,k} \geq 0 \quad \forall\ j \in J, k \in K, \tag{2.48}$$

where $k'(k)$ is the direct precedent scenario of k in the scenarios tree and t_k is the time period corresponding of scenario k. In this model it is possible to adjust the setup and production in each time period.

Robust optimization can be an alternative modeling approach to stochastic programming. In this case, the demand uncertainty is modeled in a polyhedral convex set, as follows:

$$U = \left\{ \mathbf{D} \in \mathbb{R}_+^{|J|\times|T|} |\ \xi_{j,t}^d \in [-1,1],\ \sum_{\tau=1}^{t} |\xi_{j,\tau}^d| \leq \Gamma_{j,t},\ \forall j \in J,\ t \in T \right\}. \tag{2.49}$$

in which $\xi_{j,t}^d = \left(\tilde{d}_{jt} - d_{j,t}\right)/\hat{d}_{jt}$ is the scaled demand deviation and $\tilde{d}_{j,t}$ is the random demand variable in the bounded interval $[d_{j,t} - \hat{d}_{j,t}, d_{j,t} + \hat{d}_{j,t}]$. $\hat{d}_{j,\tau}$ is the variability level that limits the maximum deviation and the parameter $\Gamma_{j,t}^d$ is the budget of uncertainty that controls the level of robustness.

Using the uncertainty set U, the worst-case realization of the demand is introduced into the following demand balance constraints:

$$H_{j,t} \geq h^{+}_{j,t} \cdot I_{j,t} = h^{+}_{j,t} \cdot \left[I^{+}_{j,0} + \sum_{\tau=1}^{t} \sum_{n \in N_\tau} q_{j,t} - \min_{\tilde{d} \in U} \sum_{\tau=1}^{t} (d_{j,\tau} + \hat{d}_{j,\tau} \cdot \xi^{d}_{j,\tau}) \right], \ \forall j \in J,\ t \in T, \quad (2.50)$$

and

$$H_{j,t} \geq h^{-}_{j,t} \cdot (-I_{j,t}) = h^{-}_{j,t} \cdot \left[I^{-}_{j,0} - \sum_{\tau=1}^{t} \sum_{n \in N_\tau} q_{j,t} + \max_{\tilde{d} \in U} \sum_{\tau=1}^{t} (d_{j,\tau} + \hat{d}_{j,\tau} \cdot \xi^{d}_{j,\tau}) \right], \ \forall j \in J,\ t \in T. \quad (2.51)$$

By applying the known robust optimization transformations, the robust counterpart model remains tractable (Alem et al., 2016):

$$\text{minimize} \quad \sum_{j=1}^{|J|} \sum_{t=1}^{|T|} (s_j x_{j,t} + H_{j,t}) \quad (2.52)$$

subject to

$$H_{j,t} \geq h_j(I_{j,0} + \sum_{\tau=1}^{t} q_{j,\tau} - \sum_{\tau=1}^{t} d_{j,\tau} + $$
$$+\Gamma^{d}_{j,t} \lambda^{d}_{j,\tau} + \sum_{\tau=1}^{t} \mu^{d}_{j,\tau,t}) \quad \forall\ j \in J, t \in T, \quad (2.53)$$

$$H_{j,t} \geq h^{-}_{j}(I^{-}_{j,0} - \sum_{\tau=1}^{t} q_{j,\tau} + \sum_{\tau=1}^{t} d_{j,\tau} + $$
$$+\Gamma^{d}_{j,t} \lambda^{d}_{j,\tau} + \sum_{\tau=1}^{t} \mu^{d}_{j,\tau,t}) \quad \forall\ j \in J, t \in T, \quad (2.54)$$
$$p_j q_{j,t} \leq C_t x_{j,t} \quad \forall\ j \in J, t \in T, \quad (2.55)$$
$$\sum_{j=1}^{|J|} p_j q_{j,t} \leq C_t \quad \forall\ t \in T, \quad (2.56)$$
$$x_{j,t} \in \{0,1\} \quad \forall\ j \in J, t \in T, \quad (2.57)$$
$$\lambda^{d}_{j,t} + \mu^{d}_{j,\tau,t} \geq \hat{d}_{j,\tau} \quad \forall\ j \in J, t \in T, \tau \leq t, \quad (2.58)$$
$$\lambda^{d}_{j,t}, \mu^{d}_{j,\tau} \geq 0 \quad \forall\ j \in J, t \in T, \tau \leq t, \quad (2.59)$$
$$q_{j,t} \geq 0 \quad \forall\ j \in J, t \in T, \quad (2.60)$$

where variables $\lambda^{d}_{j,t}$ and $\mu^{d}_{j,\tau,t}$ are from the corresponding dual auxiliary problem (see Alem et al. (2016) to see all the transformations). The traditional inventory balance constraints are trivially infeasible for any possible parameter variation, so they are replaced by (2.53) and (2.54), in which the variable $H_{j,t}$ represents the item j shortage or inventory cost in period t. Moreover, the constraints (2.58) and (2.59) related to the robust counterpart should be added. The remaining constraints are similar to those of the deterministic CLSP model.

2.2.2 System uncertainty

Lot-sizing problems under system uncertainties have received less attention by the research community than the demand uncertain lot-sizing problems. However, there are some works that consider uncertainty in productions yields, costs and times. Here, we present some of the works that incorporate system uncertainty in their problems.

Most of the works using mathematical models to tackle this issues use stochastic programming. Beraldi et al. (2006) develop a multi-stage stochastic programming to solve the parallel machine lot-sizing and scheduling problem with sequence-dependent set-up costs and uncertain processing times and apply a fix-and-relax heuristic. Huang and Küçükyavuz (2008) also propose a multi-stage stochastic model to solve an uncapacitated lot-sizing problem with random lead times, demand and costs. Moreover, a specific dynamic programming algorithm is developed to solve the model. Zhou and Guan (2010) address the uncapacitated lot-sizing problem with uncertainty in costs for a single item using two-stage stochastic model focusing on chemical industries.

Dynamic programming approaches were also used to tackle specific non-deterministic lot-sizing problems. Guan and Liu (2010) study the computational complexity of the scenario-tree lot-sizing formulations. They develop a dynamic programming framework for the stochastic single product lot-sizing problem with backlogging and/or varying capacities with polynomial time algorithms. In the model, uncertainty is considered in demand, capacity and costs. Guan (2011) uses dynamic programming to solve stochastic lot-sizing models with uncertain demand, capacity and costs. Jiang and Guan (2011) also resort to dynamic programming to solve the uncapacitated lot-sizing problem with single item and random lead times.

In non-linear approaches, Bollapragada and Rao (2006) develop a non-linear stochastic model with a specific heuristic and compare it with a simulation-based optimization method. The model considers demand and supply uncertainty for a single item, finite planning horizon and limited capacity setting.

Besides mathematical programming models, other alternative approaches are widely used. For instance, Grubbström and Wang (2003) resort to a Laplace transform and input–output analysis to model capacity-constrained production–inventory systems and use dynamic programming as solution technique. Dolgui and Prodhon (2007), Yano and Lee (1995) and Aloulou et al. (2014) describe other several works that apply simulation, artificial intelligence, analytical models, fuzzy programming, EOQ models and other approaches to establish optimal policies and quantities.

2.2.2.1 Conceptual models

In this section, we develop two conceptual models to illustrate how system uncertainty in lot-sizing can be incorporated into both stochastic programming and robust optimization models. In the two-stage stochastic model below we consider uncertainty in processing times. In the model, the setup decisions are taken in the first stage and the second-stage variables are the production quantity, the inventory and backlog decisions:

$$\text{minimize} \quad \sum_{j=1}^{|J|}\sum_{t=1}^{|T|} s_j x_{j,t} + \sum_{j=1}^{|J|}\sum_{t=1}^{|T|}\sum_{k=1}^{|K|} \pi_k (h_j I_{j,t,k} + h_j^- I_{j,t,k}^-) \tag{2.61}$$

subject to

$$I_{j,t,k} - I_{j,t,k}^- = I_{j,t-1,k} - I_{j,t-1,k}^- + q_{j,t,k} - d_{j,t} \quad \forall\, j \in J, t \in T, k \in K, \tag{2.62}$$

$$p_{j,k} q_{j,t,k} \le C_t x_{j,t} \quad \forall\, j \in J, t \in T, k \in K, \tag{2.63}$$

$$\sum_{j=1}^{|J|} p_{j,k} q_{j,t,k} \le C_t \quad \forall\, t \in T, k \in K, \tag{2.64}$$

$$x_{j,t} \in \{0,1\} \quad \forall\, j \in J, t \in T, \tag{2.65}$$

$$I_{j,t,k}, q_{j,t,k} \ge 0 \quad \forall\, j \in J, t \in T, k \in K, \tag{2.66}$$

where the parameter $p_{j,k}$ is the processing time of item j in the scenario k, the variable $q_{j,t,k}$ represents the production quantity of item j at period t in the scenario k. Moreover, the objective function and the constraints (2.62) and (2.63) are adjusted to the new variables. The remaining constraints, variables and parameters are the same of the deterministic CLSP.

To incorporate uncertainty in processing time through robust optimization, we first need to formulate the following polyhedral convex set:

$$U^p = \left\{ \mathbf{P} \in \mathbb{R}_+^{|J| \times |T|} \mid \xi_{j,t}^p \in [-1,1],\ \sum_{j=1}^{|J|} |\xi_{j,t}^p| \le \Gamma_t^p,\ \forall\, t \in T \right\}. \tag{2.67}$$

in which $\xi_{j,t}^p = \left(\tilde{p}_{j,t} - p_j\right)/\hat{p}_{j,t}$ is the scaled processing time deviation and $\tilde{p}_{j,t}$ is the random processing time variable in the bounded interval $[p_j - \hat{p}_{j,t}, p_j + \hat{p}_{j,t}]$. $\Gamma_{j,t}^p$ is the budget of uncertainty that controls the size of uncertainty set and reflects risk preferences. Parameter $\hat{p}_j$ controls the variability level or the deviation interval.

Similarly with the previous robust optimization model and based on the uncertainty set U^p, the worst-case realization of the processing time is incorporated into the capacity constraints:

$$\max_{\tilde{p} \in U^p} \left\{ \sum_{j=1}^{|J|} \tilde{p}_j q_{j,t} \right\} \le C_t,\ \forall t \in T. \tag{2.68}$$

The counterpart model is then formulated using basic robust optimization transformations:

$$\text{minimize} \quad \sum_{j=1}^{|J|} \sum_{t=1}^{|T|} (s_j x_{j,t} + h_j I_{j,t}) \tag{2.69}$$

subject to

$$I_{j,t} - I^-_{j,t} = I_{j,t-1} - I^-_{j,t-1} + q_{j,t} - d_{j,t} \forall\ j \in J, t \in T, \tag{2.70}$$

$$p_j q_{j,t} \leq C_t x_{j,t} \qquad \forall\ j \in J, t \in T, \tag{2.71}$$

$$\sum_{j=1}^{|J|} p_j q_{j,t} + \Gamma_t^a \lambda_t + \sum_{j=1}^{|J|} \mu_{j,t} \leq C_t \qquad \forall\ t \in T, \tag{2.72}$$

$$\lambda_t + \mu_{j,t} \geq \hat{p}_j q_{j,t} \qquad \forall\ j \in J, t \in T, \tag{2.73}$$

$$x_{j,t} \in \{0,1\} \qquad \forall\ j \in J, t \in T, \tag{2.74}$$

$$\lambda_t, \mu_{j,t} \geq 0 \qquad \forall\ j \in J, t \in T, \tag{2.75}$$

$$I_{j,t}, I^-_{j,t}, q_{j,t} \geq 0 \qquad \forall\ j \in J. \tag{2.76}$$

Similarly to the previous robust model, the variables λ_t and $\mu_{j,t}$ are from the corresponding dual auxiliary problem (see Alem et al. (2016) for more details). The constraints (2.73) and (2.75) related to the robust counterpart are added and the term $(\Gamma_t^a \lambda_t + \sum_{j=1}^{|J|} \mu_{j,t})$ is incorporated in constraints (2.72). The remaining constraints are similar to the deterministic CLSP model.

2.2.3 Rolling horizon approach

Uncertainty in long planning horizons in lot-sizing problems can also be tackled using rolling horizon heuristics or similar approaches. Here, we describe a few works that contributed in this research line.

Bookbinder and Tan (1988) were the first to classify the uncertainty incorporated in lot-sizing models in three strategies. The first one is “static uncertainty”, where all the decisions are made in the first period. In the “dynamic uncertainty”, the decisions of the subsequent periods can be made at each period. Finally, the “static-dynamic” strategy is a mix of both previous strategies, in which the planning horizon is divided and “dynamic uncertainty” strategy is applied at each first period of the divided planning horizon sets. Moreover the authors establish a conceptual comparison of such strategies with rolling horizon planning, stating that the “static uncertainty" is a more suitable strategy within a rolling horizon environment.

Baker (1977) was one of the first to study the effectiveness of rolling horizon in production planning. Since then, many authors showed the effectiveness in using a rolling horizon approach for long-term production planning. Recently, several works apply rolling horizon heuristics for a wide variety of problems. De Araujo et al. (2007) apply a mixed integer programming model with relax-and-fix heuristic and rolling horizon to solve the general lot-sizing and scheduling problem. Beraldi et al. (2008) also develop a rolling horizon heuristic with fix-and-relax for multi parallel machine and scheduling. Li and Ierapetritou

(2010) resort to rolling horizon to integrate planning and scheduling decisions with limited production capacity. Tolio and Urgo (2007) apply a rolling horizon approach with two-stage stochastic programming in order to tackle uncertainty in planning production and outsourcing. More recently, Bredström et al. (2013) combine robust optimization with a rolling-horizon procedure to address demand uncertainty in production planning.

2.2.4 Comparison of uncertainty modeling approaches

The incorporation of uncertainty in mathematical linear model was first proposed by Dantzig (1955). This work also proved the convexity of stochastic models for the general m number of stages. Since then, many stochastic programing models have been used to tackle uncertainty in supply chain planning problems.

In the last subsections we presented stochastic programing models to address demand and system uncertainties. The stochastic programming models are usually classified in two groups: two-stage stochastic programming and multi-stage stochastic programming. In the two-stage stochastic models the uncertainty of all periods is revealed at once at the second stage of the model. In the multi-stage stochastic models the uncertainty is revealed at each correspondent period. Stochastic models are often recognized to be computationally intractable, mainly when the number of scenarios or stages are high, which may require alternative solution methods in order to achieve good solutions.

Robust optimization (RO) models were also introduced in past subsections. Recently, it became an alternative approach to model uncertainty. Its main advantage is that the robust version of the models can be tractable, depending on how uncertainty is modeled. Moreover, it is flexible to model several problems and allows to establish trade-offs between performance and robustness (Bertsimas et al., 2011). There are several ways to establish the uncertainty set for the RO models and the level of robustness. Moreover, and because of their tractability in some cases, conventional solution techniques can be sufficient to solve the RO models.

Although the application of many modeling approaches to incorporate uncertainty in lot-sizing models, there is a lack of methodology to compare and assess the models. Sahinidis (2004) makes an exhaustive literature review of optimization under uncertainty. The author classifies the uncertainty optimization techniques in stochastic programming, fuzzy mathematical programming and stochastic dynamic programming and reiterates the need of a systematic comparison between the different uncertainty modeling approaches.

Wang (2008) uses a simulation method to empirically compare the performance of stochastic and robust modeling approaches. The simulation is similar to a Monte Carlo method, in which the models are solved and the variables before uncertainty are fixed, after that the uncertain coefficients and parameters are revealed and the costs calculated. With this simulation approach it is possible to calculate the average, standard deviation and percentiles in terms of costs. This approach may be the beginning of a systematic comparison methodology between uncertainty modeling techniques.

2.3. Integrating lot-sizing

As explained before, taking independently supply chain planning decisions for each problem may result in lower solution effectiveness (Thomas and Griffin, 1996). Lot-sizing problems can be integrated with other relevant problems in order to achieve better decision coordination and optimum solutions among the problems. In this section, we discuss the main works focusing on integrating production lot-sizing with procurement and distribution planning and also with other production planning decisions. The first subsection addresses the integration of lot-sizing with other problems in a deterministic setting. In the second subsection, we focus on integrating lot-sizing under uncertainty.

2.3.1 Integrating lot-sizing in a deterministic setting

Several works address the lot-sizing integration issue with other planning decisions in a deterministic setting, mainly focusing on the integration with distribution decisions. In this subsection, we describe the relevant works done in this field that may contribute for our research.

2.3.1.1 Integrating lot-sizing with procurement planning

The integration of production and procurement decisions was addressed by less works when compared to the other integration problems. Goyal and Deshmukh (1992) makes a literature review of integrated procurement-production systems. The authors focus on different categories of models, such as the number of products and the planning horizon. Since most of the works in integrating production and procurement planning assume static demand and infinite planning horizon, EOQ and similar modeling approaches are mainly used to solve the procurement-production decisions.

Goyal and Deshmukh (1992) also make important remarks about the models assumptions in procurement-production models that may be relevant for this research in order to build more realistic models. The main remarks are: the models do not take into account important aspects of production system, such as scheduling; also many models consider unlimited capacity and they account for only a single facility, which may be far from real case. Finally, the authors state that one of the main challenges is to incorporate probabilistic demand and probabilistic costs, which may improve the application of integrated production-procurement models.

In a different approach, Thomas and Griffin (1996) address the buyer-vendor coordination problem for the procurement stage. Different from the regular EOQ, in buyer-vendor coordination the order quantities are decided focusing on savings for both purchaser and vendor simultaneously. Thomas and Griffin (1996) describe important works that address buyer-vendor coordination. In these works, infinite horizon and general conditions are considered in order to establish optimal lot size orders.

Although there are only a few works in integrating production and procurement planning, there are relevant works that focus on order lot-sizing and supplier selection. Aissaoui et al. (2007) make an extensive literature review addressing supplier selection and order lot-sizing modeling. According to the authors, some of the gaps are: the development of algorithms for complex lot-sizing and supplier selection problems, such as for the setting with multi-product under different incentives; the consideration of uncertainty parameters to design a reliable supply chain; and the consideration of decision coordination for the supplier selection issue.

2.3.1.2 Integrating lot-sizing within hierarchical production planning

There are cases in which lot-sizing decisions can be incorporated in tactical and strategic planning (Jans and Degraeve, 2008) or integrated with production scheduling in order to achieve better coordinated decisions and reduce the overall cost. In this section we detail the relevant research that integrates lot-sizing with other production hierarchical decisions.

Integration of lot-sizing with scheduling decisions has been widely studied. Drexl and Kimms (1997) make an extensive literature review on lot-sizing and scheduling models, in which the integration of scheduling and lot-sizing is addressed. Maravelias and Sung (2009) also make a review of integrating production planning and short-term scheduling more focused on chemical industries, moreover the authors describe the importance of this integration and present modeling approaches for it. Jans and Degraeve (2008) describe some extensions of lot-sizing with scheduling decisions, such as lot-sizing with job shop scheduling, moreover they state that the "boundaries between lot-sizing and scheduling are fading" and research on the integration of lot-sizing and sequencing "constitutes a challenging research track". More recently, Guimarães et al. (2014) classify several lot-sizing and scheduling modeling approaches and formulate a new efficient model.

One of the most known problems that integrates lot-sizing and scheduling decisions is the GLSP. In this problem a number of micro-periods per period is defined a priori, and in each of them only one product can be produced while respecting the production capacity of a period (Drexl and Kimms, 1997). Other alternative models are described by Guimarães et al. (2014), in which the models are classified in two major groups: product-oriented and sequence-oriented. In product-oriented models, the model defines the sequence of products, while in sequence-oriented models there are predefined sequences from a set that the model should choose. Lot-sizing and scheduling decisions are also modeled in multi-level and multi-product settings (Drexl and Kimms, 1997) or combined with other problems, such as job-scheduling (Jans and Degraeve, 2008).

In terms of solution and modeling approaches, Maravelias and Sung (2009) provide several modeling approaches and solution strategies for lot-sizing and scheduling: detailed scheduling models, relaxed and aggregated formulations and rolling horizon approaches. Moreover, solution techniques are highlighted (for example, hierarchical methods, iterative and decomposition methods). In this sense, Guimarães et al. (2014) also point out opportunities for extending the proposed formulations to a more complex and real-world perspective, such as parallel-machine and multi-level production.

Finally, Maravelias and Sung (2009) present research opportunities for the integration of production planning and scheduling decisions. According to the authors, the main challenges of this field are in the formulation of effective scheduling models, development of iterative and hybrid computational efficient methods, incorporation of uncertainty in the models and data integration.

2.3.1.3 Integrating lot-sizing with distribution planning

We base this review on some recent works and on the literature reviews of Sarmiento and Nagi (1999), Thomas and Griffin (1996) and Fahimnia et al. (2013). The work reviewed considers finite planning horizon, dynamic demand and the use mainly of mathematical modeling approaches for formulating the problems.

Substantial costs savings can be achieved with the integration of production and distribution planning decisions. Martin et al. (1993) describe a real case study, in which the benefits of a large scale linear programming that integrates production, distribution and inventory decisions in a tactical-operational perspective are shown.

The integration of production-distribution planning has multiple perspectives. For instance, Bilgen and Günther (2010) integrate the production and distribution planning through a block planning approach for the fast moving consumer goods industry using mixed-integer programming models to minimize production and transportation costs. Fumero and Vercellis (1997) develop a model solved with Lagrangian decomposition that integrates production, machine assignment and distribution planning among different manufacturing facilities. Haq et al. (1991) formulate an integrated production-inventory-distribution model in order to minimize the total system cost considering several warehouses and retailers' facilities. The function encompasses backlog, recycling, transportation and inventory costs and the model was applied in a chemical industry. Ishii et al. (1988) develop a methodology to minimize dead stock and stock-outs when there are one manufacturing, one inventory and one retailer facility with unlimited production and transportation capacity.

Recently, several models were developed incorporating specific industries characteristics, such as product perishability issues. Amorim et al. (2012) perform a study considering perishable multi-product, multi-period and parallel production lines. Moreover, a multi-objective framework is considered to also maximize the freshness of the products delivered. Other authors, such as Yan et al. (2011), develop integrated production–distribution models for deteriorating products considering constant demand and a constant production rate with the objective of determining optimal policies to minimize the total system costs.

The combination of lot-sizing and vehicle routing problems also seems to be widely addressed in the literature. Chandra and Fisher (1994) perform a computational study in order to compare an integration of lot-sizing and vehicle routing decisions with the same decisions decoupled. Fumero and Vercellis (1999) also develop a model of production lot-sizing with vehicle routing to integrate production, inventory and distribution schedules, the model is solved through a heuristic Lagrangian relaxation.

Some authors approach the production-distribution problem using non-linear mathematical programming models. Benjamin (1989) develops a non-linear model incorporating inventory, distribution, production and shortage costs for one production plant and one retailer. To solve the model the authors use a commercial solver and a specific heuristic to show significant savings in coordinating production and distribution decisions. More recently, Chen et al. (2009) tackle the integration of production scheduling and vehicle routing for perishable products with a mixed-integer non-linear programming model solved with a heuristic algorithm.

In addition, Fahimnia et al. (2013) review several works that use different approaches in order to solve the production-distribution problem. Besides mathematical models, the main alternative solution approaches are heuristics, genetic algorithms and simulation.

In the future trends of production-distribution models, the main literature reviews (Fahimnia et al., 2013; Bilgen and Ozkarahan, 2004) reiterate that it is necessary to expand the objectives to be tackled beyond costs or profit. Hence, measures such as, service level should be incorporated. Moreover, they state that it is necessary to consider uncertainty and incorporate more realistic aspects, such as several shipment modes, heterogeneous fleet, reverse logistics and detailed operational decisions. Moreover, they suggest the usage of new solution techniques, such as new emerging metaheuristics, in order to tackle real-world problems.

2.3.1.4 Integrating lot-sizing with procurement and distribution planning

There are not many works that incorporate lot-sizing decisions in the whole supply chain planning or models focusing on deterministic supply chain coordination. According to Gebennini et al. (2009), a small number of works propose models and methods that enable practitioners to optimize the whole supply chain globally.

Pal et al. (2011) propose a model for procurement, production and shipment decisions. The model considers a three-echelon supply chain and supplier order scheduling combined with a production-shipment decisions. To solve the problem the authors develop a swarm-based heuristic.

Noorul Haq and Kannan (2006) develop an integrated supplier selection and multi-echelon distribution inventory model using “fuzzy analytical hierarchy process” and a genetic algorithm. The model focuses on supplier selection, procurement, production, inventory and distribution decisions.

Other authors, such as Yılmaz and Çatay (2006), develop a mixed-integer programming (MIP) model focusing on planning and coordination of production-distribution decisions, in which it is considered raw material and delivery costs related to the supplying stage.

In a more strategic aspect, a considerable number of works address decisions of the of the whole supply chain in a more holistic perspective. For instance, Yan et al. (2003) develop a strategic model for supply chain design in a deterministic setting. Fandel and Stammen (2004) develop a model for strategic supply chain management including scheduling and focusing more on the product’s life cycles.

Also in a more strategic perspective, other researches, such as Melachrinoudis and Min (2000) and Jayaraman and Pirkul (2001), have lot-sizing incorporated in mathematical models in order to plan strategic decisions of location of facilities and warehouses and some tactical/operational decisions related to suppliers, distribution and production.

Melo et al. (2009) and Min and Zhou (2002) present several research opportunities in modeling integrated supply chain decisions. Melo et al. (2009) state that tactical/operational decisions, such as procurement, routing and transportation modes decisions are not integrated with location decisions. Aspects of postponement decisions, reverse logistics and objective functions focused on investment measures are scarce in the literature. The main research opportunities that Min and Zhou (2002) describe are the following. Usage of multi-objectives approaches to analyse joint decisions of procurement, production and inventory planning in terms of cost, level of service and lead time. Application of mathematical models to overcome conflicts of interest, such as vendors and buyers. And application of modeling on more complex supply chain networks, such as multi-echelon and multi-period.

2.3.2 Integrating lot-sizing under uncertainty

In this subsection, the main works that integrate lot-sizing problems under uncertainty are reviewed. This literature review is focused mainly on mathematical programming models and problems with dynamic demand and finite planning horizon. Table 2.1 provides a taxonomy of the works that are categorized according to the planning perspective, the problem(s) integrated, the source(s) of uncertainty, the problem structure, the modeling approach and the solution technique. The table has the following notation:

- Perspective: S (Strategic), S-T (Strategic-tactical), T (Tactical) and T-O (Tactical-operational).
- Source of uncertainty: De (Demand), Ca (Capacity), Co (Costs), Pr (Price), Yi (Yield), Pf (Production failure), Pt (Production time), LT (Lead time) and Ti (Time).
- Problem structure: S-P (Single-product), M-P (Multi-product), M-L (Multi-level), M-M (Multi-machine) and M-F (Multi-facility).
- Modeling approach: LP (Linear programming), MIP (Mixed-integer programming), SM (Stochastic Model), SPM (Stochastic programming model), TSSPM (Two-stage stochastic programming model), MSSPM (Multi-stage stochastic programming model), RO (Robust optimization), FP (Fuzzy programming), O-S (Optimization-simulation), NL (Non-linear), CCP (Chance constrained programming) and BLP (Bilevel programming).

When compared with lot-sizing deterministic integrations, the integration of lot-sizing under uncertainty is less addressed in the scientific literature. Most of the works focus on the integration of production with distribution planning and different hierarchical production planing. Stochastic programming and fuzzy programming are the major modeling approaches used. Specific heuristics and decomposition methods are also used in some cases.

To the best of our knowledge, there are no work that has reviewed integrated lot-sizing under uncertainty in a specific framework. Nevertheless, there are two reviews that are worth mention: Mula et al. (2006) and Peidro et al. (2008) that review production and supply chain planning under uncertainty and classify the works by research topic.

Moreover, Mula et al. (2006) and Peidro et al. (2008) acknowledge gaps in modeling production and supply chain planning under uncertainty and describe some research opportunities. Several of these opportunities are aligned with the previous literature reviews. The authors state that new approaches to manage the uncertainty of each company within

Problem Integrated	Work	Perspective	Source of Uncertainty	Problem Structure	Modeling Approach	Solution Technique
Hierarchical production planning	Karabuk and Wu (2003)	S-T	De, Ca	M-P, M-F	MSSPM	Commercial solver
Hierarchical production planning	Leung et al. (2006)	T	De, Ca	M-P, M-F	TSSPM	-
Hierarchical production planning	Leung et al. (2007)	T	De, Co	M-P, M-F	Scenario based RO	Commercial solver
Hierarchical production planning	Alonso-Ayuso et al. (2014)	T	Pr	M-F	MSSPM	Commercial solver
Hierarchical production planning	Gfrerer and Zäpfel (1995)	T-O	De	M-P	LP	-
Hierarchical production planning	Zäpfel (1996)	T-O	De	M-P	LP	-
Hierarchical production planning	D. Kira (1997)	T-O	De	M-P	SPM	Commercial solver
Hierarchical production planning	Meybodi and Foote (1995)	T-O	De, Pf	M-M, M-P	TSSPM	Scheduling heuristic and rolling horizon approach
Hierarchical production planning	Balasubramanian and Grossmann (2004)	T-O	De	M-P	MSSPM, TSSPM	Shrink horizon approach
Hierarchical production planning	Sand and Engell (2004)	T-O	De, Ca, Ti, Yi	M-P	TSSPM	Lagrangian decomposition
Hierarchical production planning	Beraldi et al. (2006)	T-O	Pt	M-M, M-P	MSSPM	Fix-and-relax heuristic
Hierarchical production planning	Wu and Ierapetritou (2007)	T-O	De, Pr	M-M, M-P	MSSPM	Rolling horizon approach
Hierarchical production planning	Ramezanian and Saidi-Mehrabad (2013)	T-O	De, Pt	M-M, M-P	MIP and CCP	Rolling horizon approach and metaheuristics
Hierarchical production planning	Alem and Morabito (2012)	T-O	Co, De	M-P	RO	Commercial solver
Hierarchical production planning	Alem and Morabito (2013)	T-O	Ti, De	M-P	TSSPM	Commercial solver
Distribution planning	Chien (1993)	T	De	S-P	SM	Monte Carlo simulation
Distribution planning	Pyke and Cohen (1994)	T	De	M-P	SM	Specific algorithm
Distribution planning	Rota et al. (1997)	T	De	M-L, M-P	MIP with forecast approach	Rolling horizon approach
Distribution planning	McDonald and Karimi (1997)	T	De	M-P, M-F	LP with safety stocks	Commercial solver
Distribution planning	Escudero et al. (1999)	T	De, Supply Co, LT	M-P, M-L	TSSPM	-
Distribution planning	Gupta et al. (2000)	T	De	M-P, M-F	TSSPM, CCP	Commercial solver
Distribution planning	Yu and Li (2000)	T	De, Co	M-P, M-F	Scenario based RO	-
Distribution planning	Lee et al. (2002)	T	Ca	M-P, M-F	O-S	Iterative optimization-simulation approach
Distribution planning	Lee and Kim (2002)	T	Pt	M-P, M-F	O-S	Iterative optimization-simulation approach
Distribution planning	Agrawal et al. (2002)	T	De	M-P, M-F	TSSPM	Commercial solver
Distribution planning	Gupta and Maranas (2003)	T	De	M-P, M-F	TSSPM	Commercial solver
Distribution planning	Chen and Lee (2004)	T	De, Pr	M-P, M-F	FP	Commercial solver
Distribution planning	Ryu et al. (2004)	T	De, Ca	M-P, M-F	BIP	-
Distribution planning	Demirli and Yimer (2006)	T	Co	M-L, M-F	FP	Commercial solver
Distribution planning	Aliev et al. (2007)	T	De, Ca, Pt	M-P, M-F	FP	Metaheuristic
Distribution planning	Liang (2008)	T	Co, Ti	S-P, M-F	FP	Simplex method within an iterative algorithm
Distribution planning	Bilgen (2010)	T	Ca, Co	M-P, M-F	FP	Commercial solver
Distribution planning	Safaei et al. (2010)	T	LT, Pf, Pt	M-P, M-F	O-S	Iterative optimization-simulation approach
Procurement and distribution planning	Coronado (2007)	T	Suppliers Ca	M-L, M-F	NL - TSSPM	Iterative heuristic
Procurement and distribution planning	Von Lanzenauer and Pilz-Glombik (2002)	T	De	M-P, M-F	SPM	Commercial solver
Procurement and distribution planning	Lababidi et al. (2004)	T	De, Pr, Co, Yi	M-P, M-F	TSSPM	Commercial solver
Procurement and distribution planning	Peidro et al. (2009)	T	Supply, De, Ti, Ca, Co, LT	M-P, M-F	FP	Commercial solver
Procurement and distribution planning	Kanyalkar and Adil (2010)	T	De	M-P, M-F	Scenario based RO	Commercial solver
Strategic supply chain planning	Aghezzaf (2005)	S	De	S-P, M-F	Scenario based RO	Lagrangian relaxation
Strategic supply chain planning	Guillén et al. (2005)	S	De	M-P, M-F	TSSPM	Commercial solver
Strategic supply chain planning	Lee and Billington (1993)	S	Supply, De, LT	M-P, M-F	SM	-
Strategic supply chain planning	Sabri and Beamon (2000)	S-T	Pt, Co, LT, De	M-P, M-F	MIP, SM	Iterative hierarchical algorithm
Strategic supply chain planning	Cohen and Lee (1988)	S-T	De, LT	M-P, M-F	SM	Hierarchical algorithm heuristic
Strategic supply chain planning	Das and Sengupta (2009)	S-T	De, LT	M-P, M-F	TSSPM	Commercial solver
Strategic supply chain planning	Koutsoukis et al. (2000)	S-T	De	M-P, M-F	TSSPM	-
Strategic supply chain planning	C. Lucas (2001)	S-T	De	M-P, M-F	TSSPM	Lagrangian relaxation
Strategic supply chain planning	Alonso-Ayuso et al. (2003a)	S-T	De, Pr, Co	M-P, M-F	TSSPM	Branch-and-fix coordination
Strategic supply chain planning	Miranda and Garrido (2004)	S-T	De	S-P, M-F	NL-MIP, Policy Model	Lagrangian relaxation
Strategic supply chain planning	Santoso et al. (2005)	S-T	Supply, Co, De, Ca	M-P, M-F	TSSPM	Benders decomposition
Strategic supply chain planning	Levis and Papageorgiou (2004)	S-T	Product success, De	M-P, M-F	TSSPM	Commercial solver and hierarchical algorithms

Table 2.1 – Taxonomy table.

the supply chain are required. According to the authors, most of the works reviewed assume a simple production structure and only one uncertainty source and complex models are mainly approached with heuristics and simulations. It seems that there is a lack of exact modelling approaches to model and solve complex production problems under uncertainty. The authors highlight the following necessities for further research: development of models that contain several types of uncertainty; development of methods to incorporate uncertainty in an integrated manner; comparison and assessment of different modelling approaches.

2.3.2.1 Integrating lot-sizing within hierarchical production planning

Efforts were made on integrating lot-sizing with more detailed production planning decisions under uncertainty sources, mostly demand. Gfrerer and Zäpfel (1995) develop a hierarchical production planning model with random demand using the concept of robust production planning. Zäpfel (1996) proposes a hierarchical model (aggregated and detailed planning level) that can be incorporated in the MRP II concept to tackle random demand. D. Kira (1997) formulate a linear stochastic programming model for the hierarchical production planning with different uncertain demand distributions. In capacity planning, Karabuk and Wu (2003) develop a multi-stage stochastic model for planning the capacity for the semiconductor industry. The uncertainty is found on capacity and demand.

Balasubramanian and Grossmann (2004) develop stochastic programming models to tackle decisions of batching and scheduling under demand uncertainty. The two-stage stochastic programming models are embedded in a shrink horizon approach to solve the multistage stochastic problem. Basically, in the shrink horizon approach, the two-stage stochastic model is solved iteratively with the decisions of the current period fixed, until decisions for the entire horizon have been fixed. For the two-stage stochastic model, the authors consider scheduling as first stage variables and demand fulfilment and inventory decisions as second stage variables.

Sand and Engell (2004) develop a two-stage stochastic programming with a Lagrangian decomposition approach in order to solve the hierarchical scheduling of flexible chemical batch processes. Uncertainty is found at capacity and demand, moreover risk-averse strategies are considered.

Lot-sizing and scheduling decisions under uncertainty were also addressed by some authors. Meybodi and Foote (1995) address production planning and scheduling under uncertain demand and production failure with a stochastic model incorporating scheduling and using a rolling horizon heuristic. Wu and Ierapetritou (2007) address the production planning and scheduling problem under uncertainty through a hierarchical approach with a rolling horizon strategy. The authors develop a multi-stage stochastic programming model to tackle demand uncertainty.

Beraldi et al. (2006) develop a multi-stage stochastic programming to solve the parallel machine lot-sizing and scheduling problem with sequence-dependent setup costs and uncertain processing times. Using a fix-and-relax heuristic the authors reach an integrality gap that is lower than 3%.

Metaheuristics were also applied in lot-sizing and scheduling problems. Ramezanian and Saidi-Mehrabad (2013) propose an MIP model with a rolling horizon heuristic and a hybrid metaheuristic for scheduling. The method is focused on solving the lot-sizing and scheduling problems under uncertain processing times and product demand. In addition, the authors demonstrate that the metaheuristic performed better than the MIP model with the rolling horizon heuristic.

The integration of lot-sizing with cutting-stock problems has not been widely approached. Alem and Morabito (2012) use robust optimization in order to model a production planning problem under uncertainty in the furniture industries. The production planning is a combination of multi-item lot-sizing and cutting-stock problems. In addition, the models proposed have uncertainty both in terms of costs and demand. In their subsequent work, Alem and Morabito (2013) propose two-stage stochastic programming models to incorporate uncertainty in setup times and demand. The models also incorporate risk averse strategies using conditional value-at-risk, minimax regret, mean-risk and restricted recourse approaches.

Focusing only on planning the production for several facilities under demand uncertainty, Leung et al. (2006) formulate a two-stage stochastic programming model. Leung et al. (2007) develop a scenario based robust optimization model to the same problem, with additional uncertain costs. More recently, Alonso-Ayuso et al. (2014) apply multistage stochastic programming to incorporate uncertain copper prices in mine planning models. The different models maximize the expected profits and consider risk averse strategies, such as Value-at-Risk and variants of Conditional Value-at-Risk.

2.3.2.2 Integrating lot-sizing with distribution planning

The integration of production and distribution planning under uncertainty was addressed by a considerable number of authors. For instance, Chien (1993) tries to maximize the expected profit of a production-distribution system under a stochastic demand. The assumptions of the problem are stationary demand, constant production rate and a single product, moreover the costs considered are shortage, inventory, transportation and production costs. The author designs an iterative procedure to achieve optimal production and distribution policies, moreover a Monte Carlo simulation validates the model. In the same line, Pyke and Cohen (1994) develop a stochastic programming model to optimize integrated production-distribution systems with uncertain demand. The authors develop a specific algorithm that is able to find near-optimal solutions.

Some works were found in integrating decisions at a tactical level, most using stochastic programming models. For instance, Gupta et al. (2000) develop a two-stage stochastic programming model in order to plan a supply chain in a mid-term horizon. The model focuses on increasing customer satisfaction and reducing costs while tackling uncertainty in the demand. Moreover, a customized solution technique together with a chance constraint approach is used. Gupta and Maranas (2003) develop a two-stage stochastic model in order to plan the production and distribution decisions in a mid-term horizon under demand uncertainty. The model has several products and production facilities and takes into account the customer service level. The authors resorted to a Monte Carlo sampling method to reduce the computational complexity and solve the model with CPLEX.

Agrawal et al. (2002) develop a two-stage stochastic programming model that manages capacity, inventory, and shipments for multiple vendors. The model aims at maximizing the expected profit and it considers demand uncertainty, distinct vendors, lead times and production flexibility. Escudero et al. (1999) develop a two-stage stochastic programming model for manufacturing, assembly and distribution planning with uncertainty in demand, delivery time and supplying costs. The model considers multi-product and a multi-level structure.

In a rolling horizon perspective, Rota et al. (1997) develop an MIP model to integrate firm orders, forecasts and suppliers in order to better coordinate production planning with other activities. In the model, the demand constraints are uncertain and a rolling horizon approach is used.

Other approaches, such as standard linear programming models and robust optimization models are also used. McDonald and Karimi (1997) formulate a linear model to plan the tactical production-distribution operations. The authors incorporate safety stocks in the model in order to tackle demand uncertainty. Yu and Li (2000) develop a general tactical robust optimization model based on scenarios. The authors give examples where production costs and demand are uncertain.

Chen and Lee (2004) resort to a mixed-integer non-linear problem (MINLP) model to tackle the production-distribution decisions. The model considers multiple conflicting objectives, uncertain demand, and product prices in a supply chain. They discretize the demand in scenarios and model the product prices as fuzzy variables. The authors used an MINLP solver in order to solve small instances.

Other authors apply optimization-simulation approaches for the production-distribution problems. Usually uncertainty is tackled on the simulation. Lee et al. (2002) propose a hybrid analytic-simulation method in order to solve the production-distribution planning in a supply chain. The problem encompasses multi-period, multi-product and multi-facility. In addition, machine and distribution capacities are considered stochastic.

According to Fahimnia et al. (2013), Lee and Kim (2002) develop one of the most generic production-distribution models. The authors develop a linear model within a optimization-simulation approach in order to integrate production and distribution decisions under uncertainty in processing and distribution times. Safaei et al. (2010) develop a hybrid mathematical-simulation model to integrate production-distribution. The authors consider a setting with multi-product, multi-period, multi-site and uncertainty is found on unexpected delays, queuing, machine failure and operation time. The proposed model is deterministic and the simulation provides the feedback of the stochastic parameters.

There is also an alternative stream that applies fuzzy programming in order to incorporate uncertainty in production-distribution problems. For instance, Bilgen (2010) proposes a fuzzy mathematical programming model with fuzzines in capacity constraints and costs for a multi-echelon problem integrating strategic decisions of production line assignments with tactical decisions of production and distribution. Demirli and Yimer (2006) also develop a fuzzy integer programming model in order to reduce the overall costs of a production-distribution system considering multi-level, multi-site and fuzzy costs.

Aliev et al. (2007) develop a fuzzy model that aggregates production and distribution planning. The model deals with multi-product and multi-facility. Moreover, uncertainty is present in demand, capacity and processing times. The authors also propose a genetic algorithm to solve the model. More recently, Liang (2008) proposes a fuzzy multi-objective linear programming model to solve an integrated production-transportation planning decision in a fuzzy environment. The model aims to optimize the three fuzzy goals: minimize production and transportation costs, minimize number of rejected items and minimize the total delivery time.

2.3.2.3 Integrating lot-sizing with procurement and distribution planning

The integration of procurement and distribution planning with lot-sizing under uncertainty has been addressed by some authors. Lababidi et al. (2004) focus on tactical procurement, production and distribution planning of a petrochemical company. The authors formulate a two-stage stochastic programming model and consider demand, market prices, raw material costs and production yields. Von Lanzenauer and Pilz-Glombik (2002) also develop a stochastic programming model to integrate procurement and distribution decisions. The model considers demand uncertainty and is applied to a modified version of the Beer Distribution Game.

More recently, Kanyalkar and Adil (2010) develop a scenario based robust optimization model to integrate production planning with procurement and distribution. The model considers demand uncertainty, multi-product and multi-plant setting. The authors demonstrate the effectiveness of the robust model solving an industrial large scale problem.

Other authors develop tactical supply chain planning models using a fuzzy modeling approach. Peidro et al. (2009), for instance, formulate a fuzzy optimization model to plan a supply chain under supply, demand and process uncertainty. In the model, procurement, distribution and production planning decisions are determined. The authors also test the effectiveness of the model with data from a real automotive supply chain.

Focusing more on the integration with procurement decisions, Coronado (2007) develops a two-stage stochastic non-linear model to tackle uncertainty at suppliers' capacity in a supply chain. In the first stage diversification and safety stock decisions are made. The author proposes a sample average approximation and a decomposition heuristic in order to solve the model efficiently. In the work, risks of stock-out are taken into account.

2.3.2.4 Integrating lot-sizing for strategic supply chain planning

With regards to strategic-tactical decisions, lot-sizing can be incorporated in models with the purpose to optimize supply chains networks and plan strategic decisions. For instance, Cohen and Lee (1988) develop a stochastic network model that combines material control, production control, inventory of finished goods and distribution network control. The model incorporates demand uncertainty and is solved with a heuristic method that decomposes the stochastic sub-models in order to optimize the overall objective. Santoso et al. (2005) also develop a stochastic programming approach for planning a supply chain network under demand and capacity uncertainty. In this work, a large-scale stochastic supply chain network problem is solved with two approaches: a sample average approximation scheme is used to generate a small number scenarios and a benders decomposition method is applied together with acceleration procedures.

Still focusing on strategic planning, Guillén et al. (2005) develop a two-stage stochastic programming model to maximize the net present value, demand satisfaction and minimize the financial risk. The model incorporates stochastic demand. First-stage decision variables relate to capacity and second-stage variables relate to production and storage. Koutsoukis et al. (2000) also develop a two-stage stochastic programming model that is incorporated into a decision support system for the strategic planning of a supply chain with uncertain demand. The model considers capacity and inventory constraints and has decisions such as production/assembly/distribution sites, number of lines at each site and operational decisions of acquisition, production and distribution. C. Lucas (2001) formulates a two-stage stochastic programming model focused to plan a supply chain network. The model takes into account demand uncertainty and considers multi-facility. The authors apply a Lagrangian relaxation algorithm in order to solve large instances of the problem.

Alonso-Ayuso et al. (2003a) formulate a two-stage stochastic programming model to solve the strategic planning of a supply chain. The model considers uncertainty in demand, price and costs and is solved with a branch-and-fix algorithmic approach. This approach coordinates the selection of branching nodes to be jointly optimized (Alonso-Ayuso et al., 2003b). Das and Sengupta (2009) develop a two-stage stochastic programming model for simultaneous strategic and tactical planning in the supply chain under several uncertainty sources, such as costs, taxes, demand and transportation lead times. The model also consider several products and facilities.

Using a multi-objective approach, Sabri and Beamon (2000) develop a model for strategic and tactical planning of the supply chain also focusing on service level. The model is divided into a deterministic strategic sub-model and a stochastic operational sub-model. The operational sub-model incorporates production, demand and delivery uncertainty sources. The multi-objective approach includes customer service level, costs and flexibility (volume and delivery) of the supply chain. The authors developed an iterative algorithm to solve both strategic and operational sub-models.

In a context of the pharmaceutical industry, Levis and Papageorgiou (2004) develop a two-stage stochastic programming model considering uncertain clinical trials for products. The model combines decisions of products portfolio, lot-sizing and capacity planning and a hierarchical algorithm is used to solve the model. Focusing more on material management in a decentralized supply chain, Lee and Billington (1993) build a stochastic programming model, in which demand, supply and lead time are uncertain.

Besides stochastic programming models, other approaches are also used to join strategic and planning decisions. Aghezzaf (2005) develops a scenario based robust optimization model to solve the strategic capacity planning and facility location under uncertain demand. Although the model is more focused on strategic decisions its core contains lot-sizing aspects. To solve the model the author develops a decomposition algorithm using Lagrangian relaxation. Miranda and Garrido (2004) develop a non-linear model incorporating the economic lot-sizing policy with uncertain demand in a facility location model. To solve the model the authors develop a Lagrangian relaxation and a sub-gradient method. Ryu et al. (2004) develop a bilevel programming model to integrate production and distribution tactical decisions under demand uncertainty.

2.4. Literature insights and gaps identified

The literature reviewed covers many aspects of lot-sizing variants, uncertainty issues and integration with other problems. We believe that this literature review brings insights for this research in three key directions. Firstly, it allowed us to understand the main modeling techniques and computational complexities of basic lot-sizing models. Secondly, it showed how integration with other problems and uncertainty can be incorporated in the models and how the solution techniques are applied in order to achieve good solutions in adequate time. Finally, it revealed that, nowadays, it is critical that models incorporate the main particularities of specific industries (e.g., product freshness, type of contracts, possibility of re-manufacturing and critical uncertainty sources) and realistic issues (e.g., several number of machines and facilities, multi-level product structures and multi-product) in order to bring high valuable contributions to practical problems. To maximize the scientific contribution of this work, we intend to align the main advances and concerns of the scientific literature to the objectives of this research.

Based on the literature review it is also possible to find some gaps that seem to have not been widely addressed by the scientific community. The first one is the impact of uncertainty sources on integrated problems. There are several papers that state the advantages of integrating supply chain stages and optimization problems. Nevertheless, there is a lack of works that perform a study on how the level of uncertainty affects the decisions in a integrated system and how integrate models can manage the uncertainty impacts. For instance, important insights could be reached in a study on how the level of uncertainty in processing times affects the production capacity planning and how different models perform under these circumstances. New works bringing high quality solutions and evaluating the different approaches to tackle integrated lot-sizing under different sources and levels of uncertainty are also an opportunity of research.

Several works have explored the integration of production-distribution systems in a tactical perspective. However, there are only a few studies that have addressed the integration of lot-sizing in tactical-operational (e.g., lot-sizing and vehicle routing problem) and strategic-tactical (e.g., lot-sizing and supplier selection, lot-sizing and capacity/facility planning) perspectives under uncertainty. Additionally, there are few works that integrate lot-sizing with other problems that may have high practical value (e.g., procurement planning, cutting problems, make-or-buy decisions).

Most of the works have addressed mainly uncertainty in demand within standard lot-sizing models. Moreover, most of the stochastic programming models developed to tackle integrated lot-sizing are limited to two-stage settings. Models that combine several uncertainty sources or take into account complex lot-sizing problems (e.g., multi-level lot-sizing, lot-sizing with parallel machines) are scarce. The development of complex models may be necessary to bring superior solutions for practical problems.

Efforts to compare different solution techniques (e.g., exact, decompositions, heuristics, hybrid, metaheuristics) for integrated lot-sizing under uncertainty can provide an understanding on how suitable each approach is for solving integrated problems.

Moreover, it seems that risk measures, such as value at risk, conditional value at risk and others are not widely studied in lot-sizing integrated models. Only Alem and Morabito

(2013) and Alonso-Ayuso et al. (2014) address this matter for integrated problems. More than just applying risk-averse strategies, it could be interesting to compare and analyse the trade-offs of these strategies in terms of the impact on the solution quality and effectiveness on reducing risk for different integrated problems.

Robust optimization approaches have not been widely used to tackle uncertainty in integrated lot-sizing problems. A research opportunity exists in studying the incorporation of uncertainty for highly complex integrated systems using robust optimization models in order to take advantage of their tractability and the unnecessity of generating scenarios to incorporate uncertainty. Moreover, a study on robust optimization focusing on assessing its different uncertainty parameters and sets in order to improve the solutions for uncertain lot-sizing integrated problems may be necessary.

With the emerging of robust optimization models, several advantages and drawbacks were established for both stochastic programming and robust optimization models. Nonetheless, there is not a standardized methodology that compares and assesses both approaches or a study providing the main advantages and drawbacks for using a specific modeling approach (RO, two-stage or multi-stage stochastic programming) or solution technique. Methods and measures that contribute in this sense may be a research opportunity.

Since Bookbinder and Tan (1988), uncertainty in lot-sizing is mostly dealt in a parallel way, when compared to other optimization problems. The classification of uncertainty in static, dynamic and static-dynamic and the usage of rolling-horizon heuristics differ from how uncertainty is classified and tackled in other optimization problems. The attempt to standardize the uncertainty classification and methods categorization may be important to take advantage of the progresses in uncertainty modeling and solution approaches from other optimization problems.

Summarizing, although there are substantial research works in integrating lot-sizing under uncertainty, there are few papers that bring insights on how the combination of problem types, modeling approaches, solution techniques, uncertainty levels and number/type of uncertainty sources impact the solutions quality and effectiveness. These opportunities are closely aligned to the primary questions and objectives of this research.

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CHAPTER 3

Integrating lot-sizing, tactical planning and supplier selection in the processed food industry under uncertainty sources

Supplier Selection for Supply Chains in the Processed Food Industry

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Abstract This paper addresses an integrated framework for deciding about the supplier selection in the processed food industry under uncertainty. The relevance of including tactical production and distribution planning in this procurement decision is assessed. The contribution of this paper is three-fold. Firstly, we propose a new two-stage stochastic mixed-integer programming model for the supplier selection in the process food industry that maximizes profit and minimizes risk of low customer service. Secondly, we reiterate the importance of considering main complexities of food supply chain management such as: perishability of both raw materials and final products; uncertainty at both downstream and upstream parameters; and age dependent demand. Thirdly, we develop a solution method based on a multi-cut Benders decomposition and generalized disjunctive programming. Results indicate that sourcing and branding actions vary significantly between using an integrated and a decoupled approach. The proposed multi-cut Benders decomposition algorithm improved the solutions of the larger instances of this problem when compared with a classical Benders decomposition algorithm and with the solution of the monolithic model

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Keywords supplier selection; production-distribution planning; perishability; disjunctive programming; Benders decomposition

3.1. Introduction

The importance of food supply chain management has been growing both at the industrial and scientific levels. The challenges faced in food supply chains are at the intersection of several disciplines and go beyond the traditional cost minimization concern. Particularly, in the process food industry, companies have to deal with higher uncertainties both upstream and downstream of the supply chain. These uncertainties are related to an ever increasing product variety, more demanding customers and a highly interconnected distribution network. This implies that companies operating in the process food industry need to manage the risk/cost trade-off without disregarding freshness, sustainability and corporate social responsibility issues (Maloni and Brown, 2006).

Effective and efficient decision support models and methods for supply chain planning are critical for this sector that is the largest manufacturing sector in Europe with a turnover of 1,048 billion euros, employing over 4.2 million people (FoodDrink Europe, 2014). It is widely acknowledged that the standard tools for supply chain management perform poorly when applied to process food industries (Rajurkar and Jain, 2011). The characteristics of food supply chains are significantly different from other supply chains. The main difference is the continuous change in the quality of raw materials - from the time they are shipped from the grower to the time they are processed at the plant, and in the quality of final products - from the time they are shipped from the plant to the time they are consumed. Ahumada and Villalobos (2009) state that food supply chains are more complex and harder to manage than other supply chains. The shelf-lives of raw, intermediate and final goods together with the strong uncertainties in the whole chain challenge a good supply chain management and planning (Ahumada et al., 2012). Despite the relevant specificities of process food industries, the consideration of perishability, customers willingness to pay and risk management at the strategic and planning levels has been seldom addressed in the literature.

The present work addresses the joint decision of choosing which suppliers to select, and the planning of procurement, production and distribution in a medium-term planning horizon. We focus on companies that process a main perishable raw material and convert it into perishable final food products. These conditions happen for instance in the dairy, fresh juices and tomato sauce industries. Within this scope we integrate strategic and tactical decisions in a common framework. We consider a setting in which companies have their plants and distribution channels well established and, therefore, the supply chain strategic decisions address the supplier selection and the related product branding. We classify the suppliers and the product branding as local or mainstream. This differentiation has already been made for the agri-business (Ata et al., 2012), but never for the food processing industry. However, there are several practical examples of the leverage than can be achieved in the customers willingness to pay by branding a product as local, and correspondingly sourcing raw materials from local suppliers (Martinez, 2010; Oberholtzer et al., 2014; Frash et al., 2014). Therefore, the demand and the list price is assumed higher for fresh products that are branded and produced with local raw materials. In contrast, a similar product with a low remaining shelf-life and produced with mainstream raw materials has a lower demand and a lower list price.

Within this context, we propose a two-stage stochastic mixed-integer programming model to tackle this supplier selection problem. In the first-stage we decide the branding of products and the quantities to be procured in advance from each supplier. In the second-stage, we decide on the produced and transported quantities as well as on the quantities procured in the spot market. Uncertainties relate to the suppliers' raw material availability, suppliers' lead time, suppliers' spot market prices and demand for final products.

The different sources of uncertainty in this supplier selection problem render the corresponding stochastic programming model hard to solve as a considerable number of scenarios have to be considered. Therefore, to solve this problem we propose a multi-cut Benders decomposition method. Moreover, to improve its convergence we test several acceleration techniques.

The remainder of this paper is as follows. Section 3.2 reviews relevant contributions in the supplier selection problem. Section 3.3 describes formally the problem and the proposed mathematical formulations. Section 3.4 is devoted to the model validation through an illustrative example. In particular, the importance of considering uncertainty, the integration of the supplier selection with tactical planning decisions, and the impact of a risk-averse strategy are discussed. Section 3.5 presents the implementation of a multi-cut Benders decomposition algorithm for this problem. Section 3.6 reports computational results for larger instances. Finally, Section 3.7 draws the main conclusions and indicates future lines of research.

3.2. Literature review

The research on supplier selection problems has been traditionally divided between the operations management community that seeks an intuitive understanding of this problem, and the operations research community that explores the advantages of structuring this decision process and unveils hidden trade-offs through the use of techniques, such as mathematical programming (De Boer et al., 2001). For a thorough review of quantitative methods for the supplier selection problem the readers are referred to Ho et al. (2010).

Most of the approaches to tackle the supplier selection problem are based on the Analytic Hierarchy Process (AHP) method to help decision makers in dealing with both uncertainty and subjectivity (Deng et al., 2014). There are also examples of works that combine AHP with other techniques, such as fuzzy linear programming (Sevkli et al., 2008). Data Envelopment Analysis (DEA) is also another widely used technique for supplier selection problems. For example, Kumar et al. (2014) propose a methodology for the supplier selection taking into consideration the carbon footprints of suppliers as an attribute of the DEA model.

The most straightforward extension to the supplier selection problem is to couple it with decisions about inventory management (Aissaoui et al., 2007). Guo and Li (2014) integrate supplier selection and inventory management for multi-echelon systems. Other works incorporate other decisions, such as the carrier selection, besides determining the ordering quantities (Choudhary and Shankar, 2014).

More recently, researchers have started to address other relevant aspects that can be studied under this general problem. Chen and Guo (2013) study the importance of supplier selection in competitive markets, and indicated that besides the more evident conclusion that dual sourcing can help to mitigate supply chain risks, strategic sourcing can also be an effective tool in approaching retail competition. Qian (2014) develops an analytic approach that incorporates extensive market data when determining the supplier selection in a make-to-order production strategy. With a more practical emphasis, Hong and Lee (2013) lay the foundations of a decision support system for effective risk-management when selecting suppliers in a spot market using measures similar to the Conditional Value-at-Risk (Rockafellar and Uryasev, 2000, 2002), such as the Expected Profit-Supply at Risk. Another relevant aspect is disruption management, especially regarding the suppliers' availability. Silbermayr and Minner (2014) develop an analytic model based on Markov decision processes in which suppliers may be completely unavailable at a given (stochastic) interval of time. Due to the complexity of the optimal ordering policies, they also derive a heuristic approach.

Uncertainty has been incorporated in supplier selection problems either through stochastic programming or simulation. Moreover, distinct sources of uncertainty and different distributions for these uncertainties have been considered. Using a hybrid simulation optimization methodology, Ding et al. (2005) are able to estimate the impact of the supplier selection on the tactical processes of the supply chain, and use this information back in the decision about which suppliers to select. Stochastic programming has proved to be a suitable methodology to address complex issues involved in supplier selection. Sawik (2013) proposes a similar model to the one presented in this paper as it is able to deal with multiple periods and it accounts for uncertainty through stochastic programming. Hammami et al. (2014) propose a model for supplier selection considering uncertainty on the currency fluctuation. Through a case-study, the authors were able to show the value of the stochastic solution when compared to the deterministic model.

In light of this discussion, the main contributions of this paper to the supplier selection literature relate to accounting for uncertainty in the lead time, the consideration of distribution decisions and the emphasis on the characteristics of processed food industries. This last point is in line with an ongoing discussion about the sustainability and profitability of local sourcing for processed food industries (Schönhart et al., 2009).

In terms of solution methods, as the complexity of our problem required the use of a more sophisticated approach rather than solving the monolithic model, we show the applicability of a multi-cut Benders decomposition method to this supplier selection problem. Moreover, we show that the valid inequalities that can be obtained from a generalized disjunctive programming formulation (Raman and Grossmann, 1994) can be used in order to tighten the Benders master problem.

3.3. Problem statement and mathematical formulations

This section describes the supplier selection problem for supply chains in the processed food industry and the mathematical models that have been developed. Let $k = 1, ..., K$ be the products that are produced in the different factories ($f \in \mathsf{F}$). To produce these products the factories have to procure raw materials from the different available suppliers ($s \in \mathsf{S}$). These raw materials are classified either as mainstream ($u = 0$) or local ($u = 1$) depending on the distance between the supplier and the customers. Notice that the focus is on a divergent production structure in which a main raw material (milk, oranges or tomatoes, for example) is transformed into several final products that vary only in the packaging or in the incorporation of small amounts of other ingredients. The planning horizon is divided into periods $t = 1, ..., T$. These periods correspond to months as we are dealing with tactical planning. After production, which can take place in regular schedules or overtime, products are transferred to retailers ($r \in \mathsf{R}$), which face an uncertain demand (D^{av}_{ktr}) that also depends on the products' age $a \in \mathsf{A_k} = \{0, ..., (SL_k - 1)\}$, where SL_k corresponds to the shelf-life of product k. Notice that raw materials also have limited age, $a \in \mathsf{A_u} = \{0, ..., (\hat{SL}_u - 1)\}$, where $\hat{SL}_u$ corresponds to the shelf-life of raw material u.

The stochastic data is initially given by continuous distributions and it is then modeled on some probability space, where V is a set of discrete scenarios with corresponding probabilities of occurrence ϕ_v, such that $\phi_v > 0$ and $\sum_v \phi_v = 1$. This discretization relates to the sampling strategy used in the computational experiments. In our two-stage stochastic program, we define the quantities to procure in advance from each supplier (s_{tsf}) and the branding strategy for each product: local ($\chi_k = 1$) or mainstream ($\chi_k = 0$) as first-stage decisions. Notice that when a product is branded as local it has to be produced only using local raw materials, whereas when a product is branded as mainstream, it is possible to use a dual sourcing strategy, and therefore procure raw materials from local and mainstream suppliers. Procured quantities in the spot market ($\bar{s}^v_{tsf}$), production quantities in regular schedules and in overtime (p^{av}_{kuft} and $\bar{p}^{av}_{kuft}$), transportation flows (x^v_{ktfr}), inventory levels of both raw materials ($\hat{w}^{av}_{utf}$) and final products (w^{av}_{ktr}), and demand satisfaction (ψ^{av}_{kutr}) are the second-stage decisions.

When making the first-stage decisions there are four sources of uncertainty to be considered. The first regards the demand for final products that is not known with certainty when negotiating contracts with suppliers and while deciding the branding strategy for each product. This reflects the real-world setting in which demand for fast moving consumer food goods is highly variable. The other three sources of uncertainty are related to the suppliers of raw materials: availability, lead time and spot price. While local suppliers have more uncertainty in the available quantities to be delivered over the planning horizon, mainstream suppliers have larger and more volatile lead times. Generally, local suppliers manage less structured operations and are harder to be engaged in risk mitigating strategies, such as production in several locations. Mainstream suppliers are by definition located in farther locations, and therefore their lead times usually contain more variability. For a thorough review on the characteristics of these two types of suppliers the readers are referred to King (2010). Regarding the spot price, it usually depends more on the negotiation undertaken and on the yields of that period than on the type of supplier. Negotiating in the spot market has a critical component of price uncertainty that is reflected in our model. Figure 3.1 illustrates the general problem framework and clarifies the connection between the formulation stages and the supply chain processes.

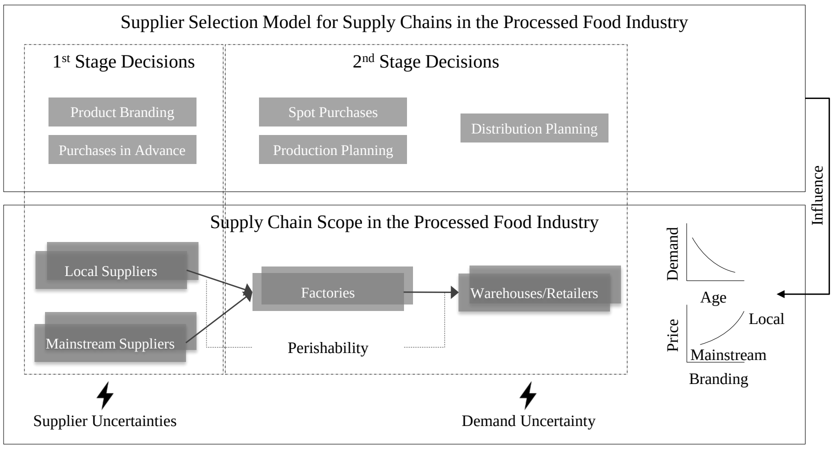

Figure 3.1 – Overview of the scope of this research.

Consider the following indices, parameters, and decision variables that are used in the stochastic programming formulation.

Indices and Sets

$k \in \mathsf{K}$	final products
$u \in \mathsf{U}$	supplier / raw material classification: 0 for mainstream, 1 for local
$s \in \mathsf{S}$	suppliers
$f \in \mathsf{F}$	factories
$r \in \mathsf{R}$	retailers
$t \in \mathsf{T}$	periods
$a \in \mathsf{A}$	ages (in periods)
$v \in \mathsf{V}$	scenarios
$\mathsf{S_u}$	set of suppliers that supply raw material of type u
$\mathsf{A_k}$	set of ages that product k may have
$\mathsf{A_u}$	set of ages that raw material u may have

Deterministic Parameters

SP_s	unit purchasing cost of raw material when bought in advance at supplier s
$TC_{sf}(\hat{TC}_{fr})$	transportation cost from supplier s (factory f) to factory f (retailer r)
$SL_k(\hat{SL}_u)$	shelf-life duration of product k (raw material u) right after being produced (time)
$HC_k(\hat{HC}_u)$	holding cost for product k (raw material u)
$PC_{kf}(\bar{PC}_{kf})$	normal (extra) production cost for product k when produced in factory f
LP_{ku}	list price for product k when branded as product of type u
E_{kf}	capacity consumption (time) needed to produce one unit of product k in factory f
$CP_{tf}(\bar{CP}_{tf})$	normal (extra) capacity of factory f available in period t

Stochastic Parameters

D_{ktr}^{av}	demand at retailer r for product k with age a in period t in scenario v
LT_{ts}^{v}	lead time offset of a shipment due to arrive in period t from supplier s in scenario v
$\bar{SP}_s^v$	unit purchasing cost of raw material when bought in a spot deal from supplier s in scenario v
AQ_{ts}^{v}	availability of raw material at supplier s for supplying in period t in scenario v
ϕ_v	probability of occurrence of scenario v

First-Stage Decision Variables

s_{tsf}	quantity of raw material procured in advance from supplier s in period t for supplying factory f
χ_k	equals 1, if product k is produced using only local raw materials (0 otherwise)
η	value-at-risk of the customer service

Second-Stage Decision Variables

τ^{av}_{tsf}	auxiliary variable that quantifies the amount of raw material procured in advance from supplier s that arrives in period t with age a for supplying factory f in scenario v
$\bar{s}^{v}_{tsf}$	quantity of raw material procured with a spot deal from supplier s in period t for supplying factory f in scenario v
$p^{av}_{kutf}(\bar{p}^{av}_{kutf})$	regular (overtime) produced quantity of product k in factory f using raw materials of type u with age a in period t and scenario v
x^{v}_{ktfr}	transported quantity of product k from factory f to retailer r in period t and scenario v
$w^{av}_{ktr}(\hat{w}^{av}_{utf})$	initial inventory of product k (raw material u) with age a in period t in scenario v at retailer r (factory f), $a = 0, ..., \min\{SL_k, t-1\}(a = 0, ..., \min\{\hat{SL}_u, t-1\})$
ψ^{av}_{kutr}	fraction of the demand for product k produced with suppliers of type u delivered with age a in period t in scenario v from retailer r, $a = 0, ..., \min\{SL_k - 1, t-1\}$
δ_v	auxiliary variable for calculating the conditional value-at-risk of the customer service

3.3.1 Mixed-integer linear programming formulation

The mixed-integer linear programming formulation of the problem is described next. The constraints that this problem is subject to are organized around the respective supply chain echelon.

3.3.1.1 Objective function

The first part of objective function (3.1) maximizes the profit of the producer over the tactical planning horizon. Expected revenue, which depends on the products' branding is subtracted by supply chain related costs: purchasing costs of raw materials, both when bought in advance and or in the spot market, holding costs for raw materials and final products, transportation costs between the supply chain nodes, and normal and extra production costs. The second part the objective function (starting with γ) maximizes the conditional value-at-risk of the customer service. This measure reflects the expected customer service of the $(1-\alpha) \cdot 100\%$ scenarios that yield the lowest customer service. For that purpose, the conditional value-at-risk of the customer service accounts for the expected customer service below a measure η (value-at-risk of the customer service) at the confidence level α. The value-at-risk of the customer service is the maximum customer service such that its probability of being lower than or equal to this value is lower than or equal to $(1-\alpha)$. This is an adaptation of the Conditional Value-at-Risk (Rockafellar and Uryasev, 2000, 2002) focusing on the customer service. Similar concepts, such as the supply-at-risk (SaR) were developed in an analogous context Hong and Lee (2013). The incorporation of risk-measures in supplier selection problems with a narrower scope have already been proposed by other authors (e.g. Sawik, 2013). The main advantage of the resulting risk-averse models is the ability to reshape the profit distribution in such a way that the worst-case scenarios are drastically reduced. Moreover, similar risk-averse models have proved to be quite effective in the food industry (Amorim et al., 2013a). The risk-aversion of the solution is controlled by weight parameter γ.

$$\max \sum_{v} \phi_v [\sum_{k,u,t,r,a} LP_{ku} \cdot D^{0v}_{ktr} \cdot \psi^{av}_{kutr} - \sum_{k,t,r,a<SL_k} HC_k \cdot w^{av}_{ktr} - \sum_{k,t,f,r} \hat{TC}_{fr} \cdot x^{v}_{ktfr}$$
$$- \sum_{k,u,t,f,a} (PC_{kf} \cdot p^{av}_{kutf} + \bar{PC}_{kf} \cdot \bar{p}^{av}_{kutf}) - \sum_{u,t,f,a<\hat{SL}_u} \hat{HC}_u \cdot \hat{w}^{av}_{utf}$$
$$- \sum_{s,f} (\bar{SP}^{v}_{s} + TC_{sf}) \cdot \bar{s}^{v}_{tsf} - \sum_{t,s,f,a} (SP_s + TC_{sf}) \cdot \tau^{av}_{tsf}] + \gamma \cdot (\eta - \frac{1}{1-\alpha} \sum_{v} \phi_v \cdot \delta_v) \tag{3.1}$$

Decision variable η retrieves an approximation of the customer service value-at-risk and the auxiliary variable δ_v is defined using Eq.(3.2). The equation defines variable δ_v making it zero when the customer service in a given scenario is higher that the customer service value-at-risk. Otherwise, variable δ_v determines the difference between the customer service value-at-risk and the corresponding mean customer service of the scenario. Figure 3.2 shows a graphical representation of the customer service conditional value-at-risk. The graph represents the distribution of the random customer service (ω). The customer service conditional value-at-risk is given by $\mathbb{E}[\omega | \omega \leq VaR_\alpha(\omega)]$, where $VaR_\alpha(\omega)$ is the value-at-risk of the customer service with a confidence of α.

$$\delta_v \geq \eta - \sum_{k,u,t,r,a \in \mathsf{A}_k} \psi^{av}_{kutr} \quad \forall v \in \mathsf{V} \tag{3.2}$$

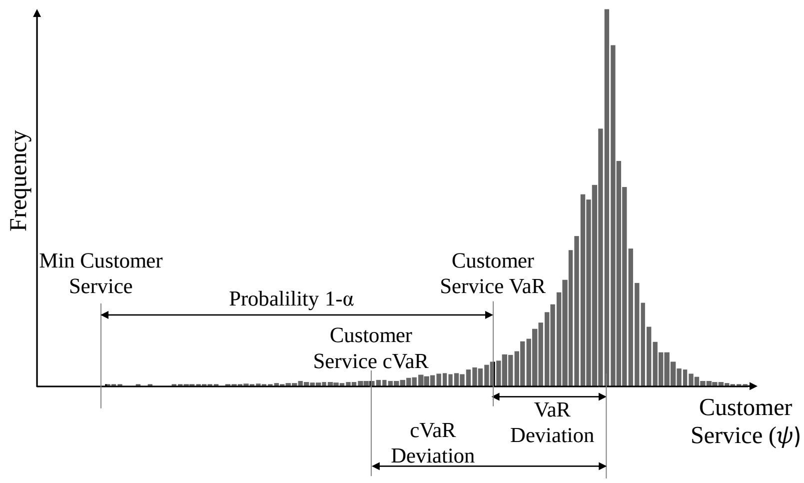

Figure 3.2 – Graphical representation of the customer service conditional value-at-risk (adapted from Sarykalin et al. (2008)).

3.3.1.2 Procurement constraints

Eq.(3.3) translates the first stage decision that defines the quantity and the arrival time of raw material from each supplier (s_{tsf}) to a second stage decision variable (τ^{av}_{tsf}) that is affected by the uncertainty on the lead time (LT^{v}_{ts}) and by the availability of the supplier (AQ^{v}_{ts}). Lead time uncertainty offsets both the arrival time and the age of the product. Therefore, if $LT^{v}_{tc} = 2$ then a product that was supposed to arrive on $t = 2$ with age 0, will arrive on $t = 4$ with age 2 in scenario v. The availability AQ^{v}_{ts} is defined a fraction between 0 and 1. The domain of variable τ^{av}_{tsf} is extended in the temporal dimension to accommodate for arrivals outside the planning horizon. The utilization of this raw material is not possible in the current planning horizon.

$$\tau^{LT^{v}_{ts},v}_{t+LT^{v}_{ts},sf} = AQ^{v}_{ts} \cdot s_{tsf} \quad \forall t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}, v \in \mathsf{V} \tag{3.3}$$

Over the entire planning horizon it is also important to enforce that the amount of product arriving with different ages is equal to the quantities that the producer has ordered discounted by the availability (3.4).

$$\sum_{t,a \in \mathsf{A_u}} \tau^{av}_{tsf} = \sum_{t} AQ^{v}_{ts} \cdot s_{tsf} \quad \forall s \in \mathsf{S}, f \in \mathsf{F}, v \in \mathsf{V} \tag{3.4}$$

Eq.(3.5) indicates that the inventory amount of raw material available to process at factory f with age 0 is equivalent to the amount bought in the spot market and bought in advance when there were no delivery delays.

$$\sum_{s \in \mathsf{S_u}} (\tau^{0v}_{tsf} + \bar{s}^{v}_{tsf}) = \hat{w}^{0v}_{utf} \quad \forall u \in \mathsf{U}, t \in \mathsf{T}, f \in \mathsf{F}, v \in \mathsf{V} \tag{3.5}$$

3.3.1.3 Production constraints

Eq.(3.6) is an inventory balance constraint for the stock of the raw materials. It also updates the age of the raw material stock and takes into account the raw materials arriving with older ages (larger than 0). Notice that the domain of the inventory variables is constrained in its definition in the beginning of Section 3.3.

$$\hat{w}^{av}_{utf} = \hat{w}^{a-1,v}_{u,t-1,f} + \sum_{s \in \mathsf{S_u}} \tau^{av}_{tsf} - \sum_{k} (p^{a-1,v}_{ku,t-1,f} + \bar{p}^{a-1,v}_{ku,t-1,f})$$
$$\forall u \in \mathsf{U}, t \in \{2, ..., T+1\}, f \in \mathsf{F}, a \in \mathsf{A_u} : a \geq 1, v \in \mathsf{V} \tag{3.6}$$

Eq.(3.7) forces the utilization of local raw material in case the product is branded as local.

$$p^{av}_{k0tf} + \bar{p}^{av}_{k0tf} \leq M(1-\chi_k) \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, f \in \mathsf{F}, a \in \mathsf{A_u} : u = 0, v \in \mathsf{V} \tag{3.7}$$

Eqs.(3.8)-(3.9) limit both normal and extra production to the available factory capacity, respectively.

$$\sum_{k,u,a \in \mathsf{A_u}} E_{kf} p^{av}_{kutf} \leq CP_{ft} \quad \forall t \in \mathsf{T}, f \in \mathsf{F}, v \in \mathsf{V} \tag{3.8}$$

$$\sum_{k,u,a \in \mathsf{A_u}} E_{kf} \bar{p}^{av}_{kutf} \leq \bar{CP}_{ft} \quad \forall t \in \mathsf{T}, f \in \mathsf{F}, v \in \mathsf{V} \tag{3.9}$$

3.3.1.4 Distribution constraints

Eq.(3.10) forces all production made in the different factories to flow to retailers within the same planning period.

$$\sum_{u,a \in \mathsf{A_u}} (p^{av}_{kutf} + \bar{p}^{av}_{kutf}) = \sum_{r} x^{v}_{ktfr} \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, f \in \mathsf{F}, v \in \mathsf{V} \tag{3.10}$$

The amount of final products entering each retailer corresponds to the inventory available to satisfy demand with age 0 (3.11). Therefore, notice that after processing the raw materials, the age of the final products is always set to 0.

$$\sum_{f} x^{v}_{ktfr} = w^{0v}_{ktr} \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, v \in \mathsf{V} \tag{3.11}$$

3.3.1.5 Demand fulfillment constraints

Eqs.(3.12)-(3.13) link the choice on the product branding as local ($\chi_k = 1, u = 1$) or mainstream ($\chi_k = 0, u = 0$) to the type of demand fulfilled that will determine the list price that the customer pays. These constraints define the revenue of the solution with the first term of the objective function (3.1).

$$\psi^{av}_{k0tr} \leq 1 - \chi_k \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, a \in \mathsf{A_k}, v \in \mathsf{V} \tag{3.12}$$

$$\psi^{av}_{k1tr} \leq \chi_k \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, a \in \mathsf{A_k}, v \in \mathsf{V} \tag{3.13}$$

Eq.(3.14) is another inventory balance constraint, but this time in the retailers premises for final products. This constraint updates the age of final products' inventory throughout the planning periods.

$$w_{ktr}^{av} = w_{k,t-1,r}^{a-1,v} - \sum_u D_{k,t-1,r}^{0v} \psi_{ku,t-1,r}^{a-1,v} \qquad (3.14)$$
$$\forall k \in \mathsf{K}, t \in \{2,...,(T+1)\}, r \in \mathsf{R}, a \in \mathsf{A_k} : a \geq 1, v \in \mathsf{V}$$

Eq.(3.15) keeps the demand fulfilled at different inventory ages below the respective demand profile (Amorim et al., 2013b).

$$\sum_u D_{ktr}^{0v} \psi_{kutr}^{av} \leq D_{ktr}^{av} \quad \forall k \in \mathsf{P}, t \in \mathsf{T}, r \in \mathsf{R}, a \in \mathsf{A_k}, v \in \mathsf{V} \qquad (3.15)$$

Eq.(3.16) ensures that the demand fulfilled with different ages is always below the demand that the customer would be willing to pay for the product in the fresher state.

$$\sum_{u,a \in \mathsf{A_k}} \psi_{kutr}^{av} \leq 1 \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, v \in \mathsf{V} \qquad (3.16)$$

$$s_{tsf}, \tau_{tsf}^{v}, \bar{s}_{tsf}^{v}, w_{ktr}^{av}, \hat{w}_{utf}^{av}, p_{kutf}^{av}, \bar{p}_{kutf}^{av}, x_{ktfr}^{v}, \psi_{kutr}^{av}, \delta_v \geq 0; \chi_k \in \{0,1\};$$
$$\eta \in \Re \quad \forall k \in \mathsf{K}, u \in \mathsf{U}, a, t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}, r \in \mathsf{R}, v \in \mathsf{V} \qquad (3.17)$$

Property 3.3.1. *The supplier selection problem for supply chains in the processed food industry (3.1)-(3.17) has complete recourse, i.e., there exists a feasible second-stage decision for every first-stage decision and independently of the uncertainties (Wets, 1983).*

3.3.2 Generalized disjunctive programming formulations

The supplier selection for supply chains in the processed food industry may be formulated with generalized disjunctive programming (GDP) (Raman and Grossmann, 1994). With GDP the different boolean decisions are represented through disjunctions. These disjunctions are then related through propositions. After having a problem formulated using GDP it is possible to derive other formulations, such as big-M (Nemhauser and Wolsey, 1988) or convex hull reformulations (Balas, 1985). For an overview of the fundamentals of GDP please refer to Grossmann and Trespalacios (2013); Castro and Grossmann (2012).

3.3.2.1 Initial GDP Formulation

One possible GDP formulation of the problem addressed in this paper is formalized next. Boolean variables Y_k indicate if product k is branded as local.

$$\max \sum_{v} \phi_v [\sum_{k,u,t,r,a} LP_{ku} \cdot D^{0v}_{ktr} \cdot \psi^{av}_{kutr} - \sum_{k,t,r,a<SL_k} HC_k \cdot w^{av}_{ktr} - \sum_{k,t,f,r} \hat{TC}_{fr} \cdot x^{v}_{ktfr}$$
$$- \sum_{k,u,t,f,a} (PC_{kf} \cdot p^{av}_{kutf} + \bar{PC}_{kf} \cdot \bar{p}^{av}_{kutf}) - \sum_{u,t,f,a<\hat{SL}_u} \hat{HC}_u \cdot \hat{w}^{av}_{utf}$$
$$- \sum_{s,f} (\bar{SP}^{v}_{s} + TC_{sf}) \cdot \bar{s}^{v}_{tsf} - \sum_{t,s,f,a} (SP_s + TC_{sf}) \cdot \tau^{av}_{tsf}] + \gamma \cdot (\eta - \frac{1}{1-\alpha} \sum_{v} \phi_v \cdot \delta_v)$$

subject to:

$$\begin{bmatrix} Y_k \\ p^{v}_{k0tf} = 0 \\ \bar{p}^{v}_{k0tf} = 0 \\ \psi^{av}_{k0tr} = 0 \\ w^{av}_{ktr} = w^{a-1,v}_{k,t-1,r} - D^{0v}_{k,t-1,r} \psi^{a-1,v}_{k1,t-1,r} \\ 0 \le D^{0v}_{ktr} \psi^{av}_{k1tr} \le D^{av}_{ktr} \\ \sum_a \psi^{av}_{k1tr} \le 1 \end{bmatrix} \veebar \begin{bmatrix} \neg Y_k \\ \psi^{av}_{k1tr} = 0 \\ w^{av}_{ktr} = w^{a-1,v}_{k,t-1,r} - D^{0v}_{k,t-1,r} \psi^{a-1,v}_{k0,t-1,r} \\ 0 \le D^{0v}_{ktr} \psi^{av}_{k0tr} \le D^{av}_{ktr} \\ \sum_a \psi^{av}_{k0tr} \le 1 \end{bmatrix} \quad \forall k \in \mathsf{K} \tag{3.18}$$

(3.2)-(3.6), (3.8)-(3.11)

$$s_{tsf}, \tau^{v}_{tsf}, \bar{s}^{v}_{tsf}, w^{av}_{ktr}, \hat{w}^{av}_{utf}, p^{av}_{kutf}, \bar{p}^{av}_{kutf}, x^{v}_{ktfr}, \psi^{av}_{kutr}, \delta_v \ge 0; \eta \in \Re$$
$$\forall k \in \mathsf{K}, u \in \mathsf{U}, a, t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}, r \in \mathsf{R}, v \in \mathsf{V} \tag{3.19}$$

$$Y_k \in \{True, False\} \quad \forall k \in \mathsf{K} \tag{3.20}$$

Disjunctions (3.18) use the information about the branding choice to set to zero part of the linear decision variables. In particular, when branding a product as local (Y_k) it is possible to set to zero both the decisions variables related with production and demand fulfillment of products using mainstream raw materials ($p^{v}_{k0tf}, \bar{p}^{v}_{k0tf}$ and ψ^{av}_{k0tr}).

3.3.2.2 Improved GDP formulation

One of the advantages of formulating a problem using GDP is the potential of deriving alternative models. The supplier selection for supply chains in the processed food industry can be addressed from a strategic level in which it is necessary to first choose between three different options: (1) use local single sourcing to produce all products; (2) use mainstream single sourcing to produce all products; (3) use dual sourcing and choose individually which product to brand as local and mainstream. The formulations that are presented next make no use of global constraints as it is possible to fit all constraints inside of the mentioned disjunctions. Boolean variables Z_i indicate if option (i) is chosen.

$$\max \sum_{v} \phi_v [\sum_{k,u,t,r,a} LP_{ku} \cdot D^{0v}_{ktr} \cdot \psi^{av}_{kutr} - \sum_{k,t,r,a<SL_k} HC_k \cdot w^{av}_{ktr} - \sum_{k,t,f,r} \hat{TC}_{fr} \cdot x^{v}_{ktfr}$$
$$- \sum_{k,u,t,f,a} (PC_{kf} \cdot p^{av}_{kutf} + \bar{PC}_{kf} \cdot \bar{p}^{av}_{kutf}) - \sum_{u,t,f,a<\hat{SL}_u} \hat{HC}_u \cdot \hat{w}^{av}_{utf}$$
$$- \sum_{s,f} (\bar{SP}^v_s + TC_{sf}) \cdot \bar{s}^v_{tsf} - \sum_{t,s,f,a} (SP_s + TC_{sf}) \cdot \tau^{av}_{tsf}] + \gamma \cdot (\eta - \frac{1}{1-\alpha} \sum_{v} \phi_v \cdot \delta_v)$$

subject to:

$$\begin{bmatrix} Z_1 \\ s_{tsf}, \bar{s}^v_{tsf}, \tau^v_{tsf} = 0 \,\forall s \in \mathsf{S}_0 \\ \hat{w}^{av}_{0fd} = 0 \\ p^v_{k0tf}, \bar{p}^v_{k0tf} = 0 \\ \psi^{av}_{k0tc} = 0 \\ \delta_v \geq \eta - \sum_{k,t,r,a \in \mathsf{A_k}} \psi^{av}_{k1tr} \\ \tau^{LT^v_{ts},v}_{t+LT^v_{ts},sf} = AQ^v_{ts} s_{tsf} \,\forall s \in \mathsf{S}_1 \\ \sum_{t,a \in \mathsf{A_u}} \tau^{av}_{tsf} = \sum_t AQ^v_{ts} s_{tsf} \,\forall s \in \mathsf{S}_1 \\ \sum_{s \in \mathsf{S}_1} (\tau^{0v}_{tsf} + \bar{s}^v_{tsf}) = \hat{w}^{0v}_{1tf} \\ \hat{w}^{av}_{1tf} = \hat{w}^{a-1,v}_{1,t-1,f} + \sum_{s \in \mathsf{S}_1} \tau^{av}_{tsf} - \sum_k p^{a-1,v}_{k1,t-1,f} \\ \sum_{k,a \in \mathsf{A_u}:u=1} E_{kf} p^{av}_{k1tf} \leq CP_{ft} \\ \sum_{k,a \in \mathsf{A_u}:u=1} E_{kf} \bar{p}^{av}_{k1tf} \leq \bar{CP}_{ft} \\ \sum_{a \in \mathsf{A_u}:u=1} p^{av}_{k1tf} = \sum_r x^v_{ktfr} \\ \sum_f x^v_{ktfr} = w^{0v}_{ktr} \\ w^{av}_{ktr} = w^{a-1,v}_{k,t-1,r} - \sum_u D^{0v}_{k,t-1,r} \psi^{a-1,v}_{k1,t-1,r} \\ D^{0v}_{ktr} \psi^{av}_{k1tr} \leq D^{av}_{ktr} \\ \sum_{a \in \mathsf{A_k}} \psi^{av}_{k1tr} \leq 1 \end{bmatrix} \vee \begin{bmatrix} Z_2 \\ s_{tsf}, \bar{s}^v_{tsf}, \tau^v_{tsf} = 0 \,\forall s \in \mathsf{S}_1 \\ \hat{w}^{av}_{1fd} = 0 \\ p^v_{k1tf}, \bar{p}^v_{k1tf} = 0 \\ \psi^{av}_{k1tc} = 0 \\ \delta_v \geq \eta - \sum_{k,t,r,a \in \mathsf{A_k}} \psi^{av}_{k0tr} \\ \tau^{LT^v_{ts},v}_{t+LT^v_{ts},sf} = AQ^v_{ts} s_{tsf} \,\forall s \in \mathsf{S}_0 \\ \sum_{t,a \in \mathsf{A_u}} \tau^{av}_{tsf} = \sum_t AQ^v_{ts} s_{tsf} \,\forall s \in \mathsf{S}_0 \\ \sum_{s \in \mathsf{S}_0} (\tau^{0v}_{tsf} + \bar{s}^v_{tsf}) = \hat{w}^{0v}_{0tf} \\ \hat{w}^{av}_{0tf} = \hat{w}^{a-1,v}_{0,t-1,f} + \sum_{s \in \mathsf{S}_0} \tau^{av}_{tsf} - \sum_k p^{a-1,v}_{k0,t-1,f} \\ \sum_{k,a \in \mathsf{A_u}:u=0} E_{kf} p^{av}_{k0tf} \leq CP_{ft} \\ \sum_{k,a \in \mathsf{A_u}:u=0} E_{kf} \bar{p}^{av}_{k0tf} \leq \bar{CP}_{ft} \\ \sum_{a \in \mathsf{A_u}:u=0} p^{av}_{k0tf} = \sum_r x^v_{ktfr} \\ \sum_f x^v_{ktfr} = w^{0v}_{ktr} \\ w^{av}_{ktr} = w^{a-1,v}_{k,t-1,r} - \sum_u D^{0v}_{k,t-1,r} \psi^{a-1,v}_{k0,t-1,r} \\ D^{0v}_{ktr} \psi^{av}_{k0tr} \leq D^{av}_{ktr} \\ \sum_{a \in \mathsf{A_k}} \psi^{av}_{k01tr} \leq 1 \end{bmatrix}$$

$$\vee \begin{bmatrix} Z_3 \\ (3.2) - (3.11) \\ (3.18) \end{bmatrix} \tag{3.21}$$

$$Z_1 \veebar Z_2 \veebar Z_3 \tag{3.22}$$

$$Z_3 \iff Y_k \veebar \neg Y_k \tag{3.23}$$

$$Z_3 \implies \vee_k [\neg Y_k] \tag{3.24}$$

$$s_{tsf}, \tau^{v}_{tsf}, \bar{s}^{v}_{tsf}, w^{av}_{ktr}, \hat{w}^{av}_{utf}, p^{av}_{kutf}, \bar{p}^{av}_{kutf}, x^{v}_{ktfr}, \psi^{av}_{kutr}, \delta_v \geq 0; \eta \in \Re$$
$$\forall k \in \mathsf{K}, u \in \mathsf{U}, a, t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}, r \in \mathsf{R}, v \in \mathsf{V} \tag{3.25}$$

$$Z_1, Z_2, Z_3 \in \{True, False\} \tag{3.26}$$

$$Y_k \in \{True, False\} \quad \forall k \in \mathsf{K} \tag{3.27}$$

Disjunctions (3.21) use the information about the sourcing strategy choice to narrow the search space. The first and the second disjunctions (Z_1 and Z_2) set to zero all variables related to mainstream sourcing / branding and to local sourcing/branding, respectively. The third disjunction (Z_3) has a embedded disjunction similar to the one presented in the previous section (cf. Section 3.3.2.1). Logic proposition (3.22) forces the choice of one of the sourcing strategies. Logic proposition (3.23) states that if a dual sourcing strategy is chosen then it is necessary to decide for each product the branding (mainstream or local). Finally, logic proposition (3.24) ensures that when choosing a dual sourcing strategy there exists at least one product that is not branded as local.

The use of GDP modeling in this context will be clearer in Section 3.5.1 where the related convex hull reformulation is used.

3.4. Model validation

In this section we validate with an illustrative example the importance of considering uncertainty, the impact of considering the integrated approach, and the effects of a risk-averse strategy in the supplier selection for supply chains in the processed food industry.

3.4.1 Instances generation

We consider a mainstream and a local supplier ($S = 2$) that supply raw material to a factory ($F = 1$). This factory converts the raw material into six products ($K = 6$) and fulfills demand for 3 retailers over a horizon of 1 year, discretized in $T = 12$ time periods. Purchasing raw materials in advance from the mainstream supplier costs 0.3 monetary units and from the local supplier it costs 0.5. Both raw materials have a shelf-life of 3 periods. All transportation costs are given in Table 3.1. Holding costs of both raw and final products are 0.05. All final products require one unit of time of capacity to be produced ($E_{kf} = 1$). There is constant available normal capacity throughout the planning horizon that is equal to the expected demand across all scenarios for products in its fresher state. Therefore, the capacity per period CP_{tf} is determined as

$$CP_{tf} = \sum_{k,r} \mathbb{E}(D^{0v}_{ktr}), \quad \forall t, f.$$

Extra capacity ($\bar{CP}_{tf}$) is 25% of the normal one. Producing within the normal capacity (PC_{kf}) costs 0.1, while using the extra capacity costs 10% more ($\bar{PC}_{kf} = 0.11$).

Origin	Destination	TC_{sf} and TC_{fr}
Mainstream Supplier	Factory	0.06
Local Supplier	Factory	0.02
Factory	Retailer 1	0.01
Factory	Retailer 2	0.02
Factory	Retailer 3	0.03

Table 3.1 – Transportation costs.

The remaining deterministic parameters for products k (SL_k and LP_{ku}) and the statistics used to generate the demand of final products (D^{av}_{ktr}) are given in Table 3.2. Note that the list price for a product branded as local is 10% higher than for a product branded as mainstream. This value is in line with the average extra willingness to pay for local products (Martinez, 2010). Demand for final products follows a gamma distribution (Van Donselaar et al., 2006). We consider that final products have a medium product quality risk, and therefore, a linear decay of demand over the age of the product until they reach zero (Amorim et al., 2013b; Tsiros and Heilman, 2005). These data reflects general parameters of perishable consumer goods products, such as milk and yogurt.

Product	SL_k	LP_{k0}	LP_{k1}	$\mathbb{E}(D^{0v}_{ktr})$	$\sigma(D^{0v}_{ktr})$
1	3	2.49	2.74	52.80	11.09
2	2	2.7	2.97	76.80	22.27
3	2	2.99	3.29	135.20	25.69
4	3	1.69	1.86	52.80	11.09
5	3	0.62	0.68	76.80	22.27
6	2	2.68	2.95	135.20	25.69

Table 3.2 – Demand related product parameters.

As already mentioned, the supply uncertainties are related to three stochastic parameters: LT^v_{ts}, $\bar{SP}^v_s$ and AQ^v_{ts}. The local supplier has no uncertainty in the delivery dates and the mainstream is characterized by an exponential negative offset (Qian, 2014) with an expected value of one period. The raw material spot cost for both suppliers is on average 10% more expensive than the corresponding cost when buying in advance (SP_s). This cost surplus follows a normal distribution (Fu et al., 2010) and has a coefficient of variation of one. Finally, the mainstream supplier has no availability issues and the local supplier has a uniformly distributed availability (Federgruen and Yang, 2008) in the interval $[0.7, 1]$. The value for α is set to 0.95 in all experiments.

3.4.2 Importance of uncertainty

To measure the importance of uncertainty we use the Expected Value of Perfect Information (EVPI) and the Value of Stochastic Solution (VSS). These two metrics are often used to evaluate the importance of using stochastic solutions over deterministic approximations.

Let RP be the optimal value of solving the two-stage stochastic programming problem (3.1)-(3.17), and consider WS_v the optimal value of solving the same problem only for scenario $v \in \mathsf{V}$. Then, the wait-and-see (WS) solution is determined as the expected value of WS_v over all scenarios. EVPI is obtained with the difference between WS and the RP:

$$\mathrm{EVPI} = \mathrm{WS} - \mathrm{RP}. \tag{3.28}$$

The EVPI may be seen as the cost of uncertainty or the maximum amount the decision maker is willing to pay in order to make a decision without uncertainty. Higher EVPIs mean that uncertainty is important to the problem (Wallace and Ziemba, 2005).

Now, let EV be the solution obtained by solving the problem in which stochastic parameters are replaced by their expected values. The expected value of using the first-stage decisions of EV over all scenarios is denoted as EEV (expected value of using the EV solution). VSS is obtained as follows:

$$\mathrm{VSS} = \mathrm{RP} - \mathrm{EEV}. \tag{3.29}$$

VSS estimates the profit that may be obtained by adopting the stochastic model rather than using the approximated mean-value one. Therefore, VSS shows the cost of ignoring the uncertainty in choosing a first-stage decision (Birge and Louveaux, 1997).

In general, there may be cases in which fixing first-stage decision variables may result in unfeasible EEV problems. However, as the supplier selection for supply chains in the processed food industry has complete recourse that is not the case (cf. Property 3.3.1).

To approximate both EVPI and VSS for the supplier selection problem we have sampled 1296 scenarios with equal probability from the stochastic parameters and solved the supplier selection problem with parameter γ set to 0 and 100. Table 3.3 presents the results for these two metrics.

	WS	RP	EEV	EVPI	VSS	EVPI/RP	VSS/RP
$\gamma = 0$	36910	36594	20657	316	15937	0.9%	43.6%
$\gamma = 100$	36910	36502	18373	408	18129	1.1%	49.7%

Table 3.3 – EVPI and VSS values for the supplier selection problem.

Both metrics are far from zero and they increase with the risk-aversion of the solution.

The importance of uncertainty grows along with the concerns about customer service. Acquiring more precise information about uncertain parameters seems not to be as critical as acknowledging the stochastic nature of this problem. The values of the EVPI show that the recourse decisions are able to correct substantially previous actions. The relative VSS values are higher than 40%, which denotes the importance of incorporating the variability of the possible outcomes instead of using expected values to make supplier selection decisions in the processed food industry context.

3.4.3 Integrated vs. decoupled approach

In order to assess the impact of considering an integrated approach to the supplier selection and production-distribution planning, we have performed sensitivity analysis on the key parameters that may influence the advantages of this approach. Through preliminary computational tests, weight γ is changed such that customer service conditional value at risk (cscVaR) is either 90% or 95%, the shelf-life of the raw materials ($\hat{SL}_u$) is varied between 3 and 9 periods and the list price of a product branded as local (LP_{k0}) is 0% or 10% higher than the mainstream list price.

Solutions are obtained with a sample average approximation scheme (Shapiro and Homem-de Mello, 1998). We sampled 81 scenarios and solved 50 instances of the approximating stochastic programming. We then evaluated the objective function by solving 1296 independently sampled scenarios. In the Decoupled approach, first problem (3.1)-(3.17) is solved without production-distribution planning constraints (3.6)-(3.11). Afterwards, having the procurement and demand fulfillment variables fixed the overall problem is solved. In the Integrated approach, problem (3.1)-(3.17) is solved simultaneously.

In Tables 3.4 and 3.5 we report for the Decoupled and Integrated approach, respectively, several indicators:

- profit - first part of objective function (3.1)
- % local - quantity of local procured raw material over the total procured raw material, $\sum(\tau^{av}_{ts'f} + \bar{s}^{v}_{ts'f}) / \sum(\tau^{av}_{tsf} + \bar{s}^{v}_{tsf}) : s' \in \mathsf{S}_1$.
- % spoiled - amount of spoiled raw material over the total procured raw material, $\sum w^{\hat{SL}_u v}_{utf} / \sum(\tau^{av}_{tsf} + \bar{s}^{v}_{tsf})$.
- % raw - total procured raw material over the total demand $\sum(\tau^{av}_{tsf} + \bar{s}^{v}_{tsf}) / \sum D^{0v}_{ktr}$.
- # local - number of products that the model chose to be branded as local, $\sum \chi_k$

Comparing the results of both approaches it is clear that the integrated approach is relevant as it is able to grasp the advantages of having a product branded as local in order to dilute key supply chain costs. These costs may arise, for example, from producing in overtime. With the 10% increase in the list price, the decoupled approach does not lead to any product being branded as local, whereas the integrated approach suggests to brand several products this way. This even happens for the case of product 4, which has an absolute difference in the list price between the two brands of less than 20 cents. These

LP_{k1}	+0%				+10%			
$\hat{SL}_u$	9		3		9		3	
cscVaR	90%	95%	90%	95%	90%	95%	90%	95%
Profit	32801	32496	33064	32866	32966	32734	33151	32920
%local	3.2%	4.2%	2.8%	3.6%	3.1%	3.5%	3.2%	6.3%
%spoiled	0.2%	0.0%	6.0%	7.4%	0.0%	0.0%	5.9%	7.4%
%raw	104.4%	107.9%	105.3%	108.5%	105.2%	107.7%	105.0%	108.8%
#local	0	0	0	0	0	0	0	0

Table 3.4 – Indicators for the decoupled approach.

LP_{k1}	+0%				+10%			
$\hat{SL}_u$	9		3		9		3	
cscVaR	90%	95%	90%	95%	90%	95%	90%	95%
Profit	35232	34834	35345	35114	35206	34863	35368	34960
%local	3.4%	3.8%	2.5%	2.9%	56.3%	66.8%	64.1%	78.5%
%spoiled	0.1%	0.1%	6.9%	6.5%	0.0%	0.0%	3.8%	3.9%
%raw	103.6%	106.0%	106.0%	106.7%	112.4%	117.3%	116.1%	121.6%
#local	0	0	0	0	3	4	4	5

Table 3.5 – Indicators for the integrated approach.

decisions force the amount of local raw material to rise considerably above 50%. This indicates that the sourcing/branding decisions in the processed food industry may need to take a wider view over the supply chain than just focusing on the procurement processes. Results also show that the logistics characteristics of local suppliers (for instance smaller and less variable lead time) may not constitute a significant attribute to rise considerably the amount of raw materials bought from such suppliers.

Taking advantage of the higher customer willingness to pay for local products increases profit and lowers the amount of spoiled raw material. The lower levels of raw material reaching their shelf-lives is related with the difference in the lead time uncertainty between mainstream and local suppliers. On the other hand, both higher service levels and lower shelf-lives of raw materials lead to an increase in the amount of spoiled material and an increase of the quantities purchased from suppliers in relation to the actual demand. The quantity of raw materials procured is also related to the amount of local supplies due to the availability uncertainties that corresponding suppliers are subject to.

Across all solutions a dual sourcing strategy is chosen. This is in line with other qualitative supplier selection studies in the process food industry that reiterate the importance of complementary in procurement (Vukina et al., 2009). Regarding the trade-off between profit and cscVaR, small losses in the average profit may lead to substantial shift in cscVaR.

3.4.4 Risk-neutral vs. risk-averse strategy

In Section 3.3.1 we have introduced in the supplier selection problem a new objective function that aims to maximize the customer service conditional value-at-risk. The behavior of such risk-aversion measures is well documented for the case in which profit is the metric to be tackled. The previous section showed the relatively low influence of attributing more weight to customer service conditional value-at-risk on the average profit. Nevertheless, as this model deals with an uncertain setting, it is relevant to go one step further and understand the impact on the profit distribution as this customer service measure is optimized.

To obtain such insights we have used the results obtained for the integrated approach when solving the 1296 independently sampled scenarios in the previous section. These results were extended by considering a risk-neutral approach ($\gamma = 0$). The risk-neutral approach yielded a customer service conditional value-at-risk of 80%. The results of the profit distribution after the uncertainty realization are similar across the instances with different shelf-lives and different list prices. Figure 3.3 shows the results for the instance in which the list price of a product branded as local (LP_{k0}) is the same as the mainstream list price and the shelf-life of the raw materials ($\hat{SL}_u$) is 3 periods. Each series correspond to a risk-aversion strategy regarding the customer service: neutral (cscVaR equals 80%), averse (cscVaR equals 90%) and very averse (cscVaR equals 95%).

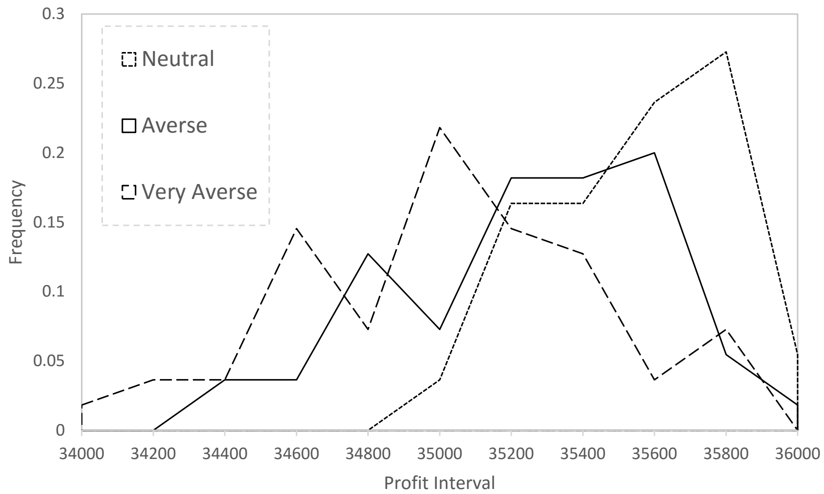

Figure 3.3 – Profit distribution for different risk-aversion strategies.

Comparing the three risk-aversion strategies it can be seen that as expected the average profit has a slight decrease as we aim to provide better service levels. But, more importantly, it seems that the dispersion of the profit increases as we aim for a more conservative attitude towards the customer service level. At a first glance this is counterintuitive because usually risk-averse strategies trade-off average profits by more predictable outcomes. However, to achieve higher service levels - averse to losses in the cscVaR, producers have to take riskier decisions, such as procuring more locally and more quantity. For certain uncertainty realization this may yield significant losses.

3.5. Multi-cut Benders decomposition algorithm

Even while using a sample average approximation scheme, it is necessary to solve a large number of two-stage stochastic programs that are not trivial to solve using monolithic models as the ones described in Section 3.3. In this Section we discuss a multi-cut Benders decomposition method that can be embedded in the sample average approximation scheme in a hybrid solution approach to solve this supplier selection problem (Santoso et al., 2005).

Benders decomposition (Benders, 1962) is a solution method that is more commonly known as L-Shaped method when applied to stochastic programming (Van Slyke and Wets, 1969). This solution method partitions the complete formulation into two models. The Benders master problem approximates the cost of the scenarios in the space of first-stage decision variables, and the Benders subproblems are obtained from the original one by fixing the first stage variables to the values obtained in the master problem. This solution method iterates between these two models improving the upper bounds obtained in the master problem (UB) with information coming from the lower bounds of the subproblems (LB).

The resulting Benders subproblems (BSP) that may be decomposed for each scenario $v \in \mathsf{V}$ are formulated for each iteration i as follows:

$$\max \sum_{v} \phi_v [\sum_{k,u,t,r,a} LP_{ku} \cdot D^{0v}_{ktr} \cdot \psi^{av}_{kutr} - \sum_{k,t,r,a<SL_k} HC_k \cdot w^{av}_{ktr} - \sum_{k,t,f,r} \hat{TC}_{fr} \cdot x^{v}_{ktfr}$$
$$- \sum_{k,u,t,f,a} (PC_{kf} \cdot p^{av}_{kutf} + \bar{PC}_{kf} \cdot \bar{p}^{av}_{kutf}) - \sum_{u,t,f,a<\hat{SL}_u} \hat{HC}_u \cdot \hat{w}^{av}_{utf}$$
$$- \sum_{s,f} (\bar{SP}^{v}_{s} + TC_{sf}) \cdot \bar{s}^{v}_{tsf} - \sum_{t,s,f,a} (SP_s + TC_{sf}) \cdot \tau^{av}_{tsf}] - \gamma \cdot (\frac{1}{1-\alpha} \sum_{v} \phi_v \cdot \delta_v)$$

subject to:

$$\delta_v \geq \eta^i - \sum_{k,u,t,r,a \in \mathsf{A_k}} \psi^{av}_{kutr} \tag{3.30}$$

$$\tau^{LT^{v}_{ts},v}_{t+LT^{v}_{ts},sf} = AQ^{v}_{ts} s^{i}_{tsf} \quad \forall t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F} \tag{3.31}$$

$$\sum_{t,a \in \mathsf{A_u}} \tau^{av}_{tsf} = \sum_{t} AQ^{v}_{ts} s^{i}_{tsf} \quad \forall s \in \mathsf{S}, f \in \mathsf{F} \tag{3.32}$$

(3.5)-(3.6)

$$p^{av}_{k0tf} + \bar{p}^{av}_{k0tf} \leq M(1-\chi^{i}_{k}) \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, f \in \mathsf{F}, a \in \mathsf{A_u} : u = 0 \tag{3.33}$$

(3.8)-(3.11)

$$\psi^{av}_{k0tr} \leq 1 - \chi^{i}_{k} \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, a \in \mathsf{A_k} \tag{3.34}$$

$$\psi^{av}_{k1tr} \leq \chi^{i}_{k} \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, r \in \mathsf{R}, a \in \mathsf{A_k} \tag{3.35}$$

(3.14) - (3.16)

$$\begin{aligned} \tau^{v}_{tsf}, \bar{s}^{v}_{tsf}, w^{av}_{ktr}, \hat{w}^{av}_{utf}, p^{av}_{kutf}, \bar{p}^{av}_{kutf}, x^{v}_{ktfr}, \psi^{av}_{kutr}, \delta_v \geq 0; \\ \forall k \in \mathsf{K}, u \in \mathsf{U}, a, t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}, r \in \mathsf{R} \end{aligned} \tag{3.36}$$

In the Benders subproblems we use the optimal first-stage solution of variables s_{tsf}, χ_k and η coming from the solution of the master problem in the previous iteration i that are denoted as $s^{i}_{tsf}, \chi^{i}_{k}$ and η^{i}.

The Benders master problem (BMP) is formulated as follows:

$$\max \gamma \cdot \eta - \sum_{v} \phi_v \cdot \theta_v \tag{3.37}$$

$$\begin{aligned} \theta_v \geq -\eta \Gamma^{vi} + \sum_{t,s,f} AQ^{v}_{ts} s_{tsf} \Delta^{vi}_{tsf} + \sum_{t,s,f} AQ^{v}_{ts} s_{tsf} \Theta^{vi}_{sf} + \sum_{k,t,f,a} M(1-\chi_k)\Lambda^{avi}_{ktf} \\ + \sum_{t,f} CP_{ft} \Xi^{vi}_{tf} + \sum_{t,f} \bar{CP}_{ft} \Pi^{vi}_{tf} + \sum_{k,t,r,a} (\Omega^{avi}_{ktr} - \chi_k \Omega^{avi}_{ktr}) + \sum_{k,t,r,a} \chi_k \Upsilon^{avi}_{ktr} \\ + \sum_{k,t,r,a} D^{av}_{ktr} \Phi^{avi}_{ktr} + \sum_{k,t,r} \Psi^{vi}_{ktr} \quad \forall v \in \mathsf{V} \end{aligned} \tag{3.38}$$

$$s_{tsf} \geq 0; \chi_k \in \{0,1\}; \eta \in \Re \quad \forall k \in \mathsf{K}, t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F}$$

In the Benders master problem we use the dual values Γ^{vi}, Δ^{vi}_{tsf}, Θ^{vi}_{sf}, Λ^{avi}_{ktf}, Ξ^{vi}_{tf}, Π^{vi}_{tf}, Ω^{avi}_{ktr}, Υ^{avi}_{ktr}, Φ^{avi}_{ktr} and Ψ^{vi}_{ktr} of constraints (3.30), (3.31), (3.32), (3.33), (3.8), (3.9), (3.34), (3.35), (3.15) and (3.16), respectively, in iteration i. Note that after preliminary experiments we chose to add an optimality cut per scenario (3.38) in the master problem instead of a single global cut (Birge and Louveaux, 1988; You and Grossmann, 2013). As mentioned before this problem has complete recourse, therefore, no feasibility cuts are necessary (cf. Property 3.3.1).

Remark 1. *It is possible to use a more intensive multi-cutting strategy by introducing cuts per time period t. However, it is necessary to distinguish the customer service in* *each period, and therefore, to rewrite Eq.(3.2) as* $\delta_v \geq \eta - \sum_{k,u,r,a \in \mathsf{A_k}} \psi^{av}_{kutr} \quad \forall t \in \mathsf{T}, v \in \mathsf{V}$. *Consequently dual values* Γ^{vi} *have to be extended to incorporate the time dimension.*

Algorithm 1 outlines the main steps of the solution method, where ε is a very small threshold value.

Algorithm 1: Outline of Benders solution method.

initialize s^0_{tsf}, χ^0_k and η^0;
set $UB = \infty$ and $LB = -\infty$;
while $UB - LB > \varepsilon$ **do**
    Solve BSP;
    Get Second-Stage Variables;
    Update UB;
    Get Duals;
    Add Optimality Cuts to BMP;
    Get s_{tsf}, χ_k, η;
    Update LB;
    Update s^i_{tsf}, χ^i_k and η^i on the BSP;

The Benders decomposition algorithm is known to have some convergence issues that can be mitigated through acceleration techniques. In the remainder of this section we discuss approaches that can be used to this end.

3.5.1 Tightening the Benders master problem

The resulting BMP from the original formulation (cf. Section 3.3.1) has no constraints besides the on-the-fly optimality cuts and the decision variables domain constraints. Therefore, before “enough” cuts are added into the BMP the convergence of the solution method is expected to be rather slow. The lack of first-stage constraints is related to two characteristics of this problem. Firstly, the uncertainty of suppliers, both in the available quantity and on the lead time, forces a translation of the purchased quantities in advance s_{tsf} into a second-stage decision variable τ^{av}_{tsf}. Secondly, the two main first-stage decision variables (s_{tsf} and χ_k) are not tightly related due to the multi-echelon scope of the supplier selection for supply chains in the processed food industry, which separates the acquisition of raw materials u from the transformation and selling of final products k.

With the GDP formulation presented in Section 3.3.2.2 we are able to tighten the first-stage decisions by introducing the three disjunctions Z_i related with the sourcing strategy. Transforming the GDP formulation into a mixed-integer programming model by applying classical Boolean algebra rules to convert the logic propositions (Williams, 1999) and reformulating the disjunctions using a hull reformulation (Lee and Grossmann, 2000) results in the following first-stage constraints that are added to the BMP.

$$z_1 \leq \chi_k \quad \forall k \in \mathsf{K} \tag{3.39}$$

$$1 - z_2 \geq \chi_k \quad \forall k \in \mathsf{K} \tag{3.40}$$

$$K - z_3 \geq \sum_k \chi_k \tag{3.41}$$

$$z_1 + z_2 + z_3 = 1 \tag{3.42}$$

$$s_{tsf} = s^2_{tsf} + s^3_{tsf} \quad \forall t \in \mathsf{T}, s \in \mathsf{S}_0, f \in \mathsf{F} \tag{3.43}$$

$$s_{tsf} = s^1_{tsf} + s^3_{tsf} \quad \forall t \in \mathsf{T}, s \in \mathsf{S}_1, f \in \mathsf{F} \tag{3.44}$$

$$0 \leq s^1_{tsf} \leq \sum_{t' \geq t} (CP_{ft} + \bar{CP}_{ft}) z_1 \quad \forall t \in \mathsf{T}, s \in \mathsf{S}_1, f \in \mathsf{F} \tag{3.45}$$

$$0 \leq s^2_{tsf} \leq \sum_{t' \geq t} (CP_{ft} + \bar{CP}_{ft}) z_2 \quad \forall t \in \mathsf{T}, s \in \mathsf{S}_0, f \in \mathsf{F} \tag{3.46}$$

$$0 \leq s^3_{tsf} \leq \sum_{t' \geq t} (CP_{ft} + \bar{CP}_{ft}) z_3 \quad \forall t \in \mathsf{T}, s \in \mathsf{S}, f \in \mathsf{F} \tag{3.47}$$

$$s^1_{tsf}, s^2_{tsf} s^3_{tsf} \geq 0; z_1, z_2, z_3, \chi_k \in \{0, 1\} \tag{3.48}$$

Note that χ_k are the binary variables resulting from transforming Boolean variables Y_k and z_1, z_2, z_3 are the binary variables resulting from transforming Boolean variables Z_1, Z_2, Z_3, respectively. Moreover, $s^1_{tsf}, s^2_{tsf}, s^3_{tsf}$ are the disaggregated variables of s_{tsf} for each disjunctive term.

3.5.2 Convex combinations

The key idea in this acceleration scheme is to consider prior solutions to the BMP, and then to modify the evaluation of the objective function to also optimize over best convex combination of multipliers (Smith, 2004).

Let s^i_{tsf}, χ^i_k and η^i for $i = 1, ..., I$ be the solutions found after solving the BMP over the last i iterations. Parameter I controls the frequency for which the modified BSP (mBSP) is solved. In this problem the solution of the first-stage decision variables s_{tsf}, χ_k and η that was found in the previous iteration is replaced by the convex combination of these variables over the past I iterations $\sum_i \lambda_i s^i_{tsf}$, $\sum_i \lambda_i \chi^i_k$ and $\sum_i \lambda_i \eta^i$, respectively. The objective function (3.30) is modified by adding the following term

$$\sum_i \lambda_i \cdot \gamma \cdot \eta^i.$$

Moreover, it is necessary to add the following constraints:

$$0 \leq \lambda \leq 1 \tag{3.49}$$

$$\sum_i \lambda_i = 1 \tag{3.50}$$

After solving mBSP in a given iteration, Algorithm 1 proceeds by getting the dual values of the subproblem constraints and adding the associated cuts to the BMP. Once mBSP is solved in one iteration, BSP is solved for the next I iterations.

3.5.3 Solving a single Benders master problem

In the classical Benders solution method, outlined in Algorithm 1, we alternate between solving a master problem and the subproblems. In this acceleration scheme we solve a single master problem and generate Benders cuts on the fly as we find feasible master solutions. This general approach is named branch-and-check (Thorsteinsson, 2001). This approach can be also seen as a branch-and-cut algorithm with the Benders subproblems sourcing the cuts. In the reminder of the paper we use modern Benders to refer to this method.

This method takes advantage of callback functions in the solver of the master problem. Its main advantage is that it avoids considerable rework in the branch-and-bound because we are keeping the same tree throughout the iterations of the Benders algorithm. Its main drawback is the harder implementation procedure.

3.6. Computational experiments

In this section we describe computational experiments using the multi-cut Benders decomposition algorithm presented in Section 3.5. We sampled 81, 256 and 625 scenarios from the instances described in Section 3.4.1 and solved for the case in which it is necessary to decide about the supplying/branding strategy of 6, 12 and 24 products. Parameter γ was set to 0 and to 1000. Therefore, in total we report results for 18 instances. All the programs were implemented in C++ and solved using IBM ILOG CPLEX Optimization Studio 12.4 on an Intel E5-2450 processor under a Scientific Linux 6.5 platform. For instances with 81 scenarios, each run was limited to 2 cores of the processor and 8GB of RAM. For instances with 12 and 24 products under 256 and 625 scenarios the execution was limited to 3 cores and 12GB of RAM.

In order to achieve better computational results, we solved the scenario subproblems with parallel computing. We grouped the scenarios into subproblems, in a way that each subproblem has 9 to 50 scenarios, depending of the number of scenarios in each instance. This method was effective in improving the computational performance and in reducing the amount of RAM required.

Table 3.6 shows the size of the monolithic model for each instance.

Products (K)	Scenarios	Constraints	Variables	Binary Variables
6	81	167,265	235,903	6
6	256	922,624	1,038,368	6
6	625	2,252,500	2,535,032	6
12	81	330,561	468,460	12
12	256	1,788,928	1,983,014	12
12	625	4,367,500	4,841,288	12
24	81	657,153	933,574	24
24	256	3,521,536	3,872,306	24
24	625	8,597,500	9,453,800	24

Table 3.6 – Size of monolithic model for each instance.

We report in Table 3.7 and 3.8 results for each instance using the mixed-integer solver to solve the monolithic model (Monolithic), modern Benders decomposition algorithm (MB) (cf. Section 5.3) and the same algorithm with the hull reformulation (MB+H) (cf. Section 5.1). In Table 3.7, instances were run with the parameter parameter γ set to 0. The results of Table 3.8 represent the runs with γ set to 1000. All solution methods were limited to 21600 seconds (6 hours). The complete results comparing the performance of all the methods are available upon request.

The results show that for the instances with 6 products and 81 scenarios CPLEX was able to solve to optimality within the given time. However, for the more realistic and larger instances, with more scenarios and products, most of the times the solver was not able to find a feasible solution to the model. The Benders algorithms did not have enough time to converge with 12 or 24 products, but they achieved better overall results in instances with 6 products and in all instances with 256 and 625 scenarios. When comparing the lower bound, modern Benders decomposition methods (with or without hull reformulation) outperformed all other methods as they were able to find reasonable or good solutions in all instances.

Although Benders decomposition achieved better solutions in almost every case, it was not able to obtain good upper bounds in instances with a larger number of products, which resulted in higher gaps. This may be caused by the structure and the size of the model, and also by the slow convergence of Benders decomposition in some cases (You and Grossmann, 2013).

When compared with classical Benders decomposition, modern Benders decomposi-

Products (K)	Scenarios		Monolithic	MB	MB+H
6	81	Lower Bound	34,625.94	34,625.94	34,625.94
		Optimality Gap	0.00%	0.00%	0.00%
		Runtime (s)	4,710	937	1,072
	256	Lower Bound	33,280.164	34,451.741	34,451.741
		Optimality Gap	9.01%	0.00%	0.00%
		Runtime (s)	21,610*	3,842	6,262
	625	Lower Bound	-	34,193.966	34,193.966
		Optimality Gap	-	0.00%	0.00%
		Runtime (s)	21,621*	16,152	17,838
12	81	Lower Bound	55,439.168	55,433.481	55,501.203
		Optimality Gap	5.14%	9.15%	9.64%
		Runtime (s)	21,606*	21,609*	21,604*
	256	Lower Bound	-	45,737.085	45,756.344
		Optimality Gap	-	17.59%	19.47%
		Runtime (s)	21,615*	21,610*	21,622*
	625	Lower Bound	-	64,207.166	64,129.093
		Optimality Gap	-	21.10%	16.33%
		Runtime (s)	21,641*	21,679*	21,677*
24	81	Lower Bound	90,341.772	90,709.587	90,728.562
		Optimality Gap	9.21%	19.38%	19.49%
		Runtime (s)	21,613*	21,609*	21,610*
	256	Lower Bound	-	95,675.549	95,642.661
		Optimality Gap	-	23.76%	20.72%
		Runtime (s)	21,633*	21,663*	21,750*
	625	Lower Bound	-	97,325.228	97,776.63
		Optimality Gap	-	27.90%	23.77%
		Runtime (s)	21,750*	22,157*	21,760*

* Execution time limite reached.
- No feasible solution found.

Table 3.7 – Results for the supplier selection problem with parameter γ set to 0.

Products (K)	Scenarios		Monolithic	MB	MB+H
6	81	Lower Bound	35,295.29	35,295.29	35,295.29
		Optimality Gap	0.00%	0.00%	0.00%
		Runtime (s)	5,538	971	1,101
	256	Lower Bound	35,060.015	35,060.015	35,060.015
		Optimality Gap	4.87%	0.00%	0.00%
		Runtime (s)	21,608*	4,733	7,202
	625	Lower Bound	-	34,880.382	34,880.382
		Optimality Gap	-	0.00%	0.00%
		Runtime (s)	21,622*	16,725	19,798
12	81	Lower Bound	56,169.77	56,169.77	56,249.42
		Optimality Gap	5.01%	8.41%	10.45%
		Runtime (s)	21,606*	21,605*	21,604*
	256	Lower Bound	-	46,273.255	46,293.80
		Optimality Gap	-	22.95%	21.43%
		Runtime (s)	21,618*	21,678*	21,614*
	625	Lower Bound	-	64,848.425	64,885.77
		Optimality Gap	-	19.15%	21.15%
		Runtime (s)	21,645*	21,691*	21,772*
24	81	Lower Bound	89,800.494	91,150.52	90,994.117
		Optimality Gap	10.63%	22.57%	21.06%
		Runtime (s)	21,609*	21,619*	21,624*
	256	Lower Bound	-	96,001.70	95,912.45
		Optimality Gap	-	24.38%	27.28%
		Runtime (s)	21,677*	21,923*	21,900*
	625	Lower Bound	-	98,215.09	97,459.08
		Optimality Gap	-	27.73%	28.35%
		Runtime (s)	21,700*	21,842*	21,931*

Table 3.8 – Results for the supplier selection problem with parameter γ set to 1000.

tion achieved better convergence in all instances. This can be explained by the faster solving time of the master problem and also by the usage of only one single exploration tree, in a way that it is not necessary to create a search tree and revisit the same nodes at each iteration.

The other acceleration techniques did not perform as well as expected. Convex combination was not able to improve the convergence and in instances with 256 and 625 scenarios, it reached the total amount of memory allowed in the first iterations. Nonetheless, we can not conclude that these methods are not effective for other models or instances. In instances with more products, the hull reformulation constraints were able to improve the optimality gap of the solutions.

The solution performance of all methods seems to decrease when the parameter γ changes from 0 to 1000. This is line with the work of Miller and Ruszczynski (2011) which shows that the more traditional decomposition algorithms have a better performance for risk-neutral models ($\gamma = 0$) rather than for risk-averse ones ($\gamma = 1000$).

Figure 3.4 shows the convergence of the upper and lower bound for the multi-cut Benders decomposition algorithm variants and the monolithic approach when solving the instance with 6 products, 81 scenarios and γ set to 0. In this case, the modern Benders method had a faster convergence than other variants of Benders and than the monolithic approach.

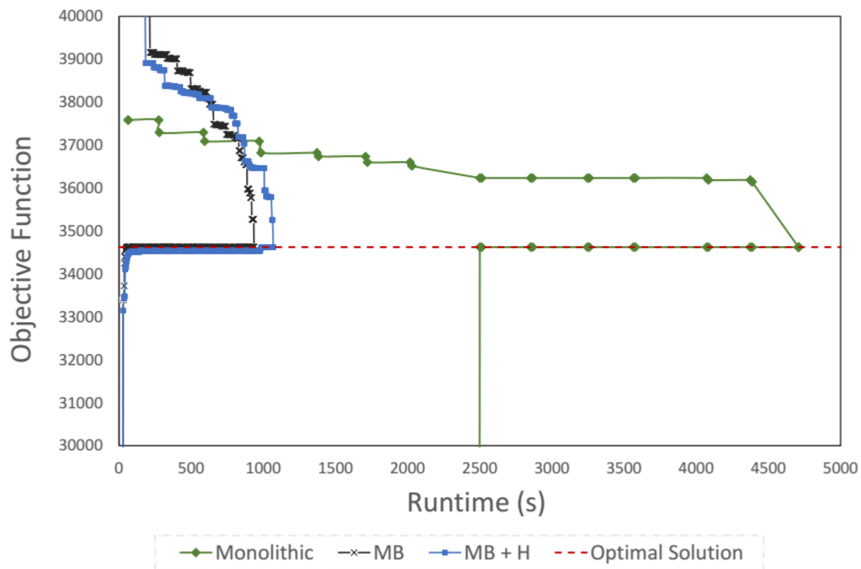

Figure 3.4 – Comparison of the convergence of the multi-cut Benders decomposition algorithm variants and the monolithic approach for the instance with 6 products, 81 scenarios and γ set to 0

3.7. Conclusions and future work

This paper proposes a novel formulation to tackle the integrated decision of supplier selection and production-distribution planning for processed food supply chains. Uncertainty is present in the suppliers' processes namely in lead time, availability and spot price, and in customers' demand, which furthermore depends on the age of the sold product. Results show that only by taking such an integrated approach of tactical and strategic levels, it is possible to make better decisions regarding sourcing of perishable raw materials to produce processed food products. The advantages of the premium price customers are willing to pay is undervalued by decoupled approaches.

Due to the difficulty in solving the problem, we explored a multi-cut Benders decomposition algorithm that leverages the different proposed formulations. This algorithm suits the structure of this supplier selection problem, however, in a short time span it is hard to obtain optimal solutions. Modern Benders decomposition was able to improve significantly the results in comparison with the classical Benders method and the monolithic model. Although acceleration techniques did not perform effectively, hull reformulation applied to the Benders master problem showed that it is potentially a promising method to improve the convergence of Benders, particularly in problems where one can take more advantage of disjunctive programming to tighten the master problem.

Future research in terms of modelling should focus on improving the realistic aspects of the models, for example by considering setup costs and the quality decay of raw materials throughout the aging process. In terms of solution methods, it would be interesting to explore other possible decomposition algorithms, such as Lagrangian decomposition and cross decomposition strategies.

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Appendix 3.A Supplementary material

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	34,625.94	34,625.94	0.00%	4,710.27	-
			CB	34,296.58	34,790.36	1.42%	21,760.88	431
			CB+H	34,375.16	35,029.88	1.87%	21,640.94	372
			CB+CC	33,610.70	35,395.69	5.04%	21,663.20	185
6	0	81	CB+H+CC	33,535.97	35,392.65	5.25%	22,131.42	210
			MB	34,625.94	34,625.94	0.00%	936.97	297
			MB+H	34,625.94	34,625.94	0.00%	1,071.81	373
			MB+CC	34,625.94	34,625.94	0.00%	8,427.81	296
			MB+H+CC	34,625.94	34,625.94	0.00%	11,223.45	368
			Monolithic	35,295.29	35,295.29	0.00%	5,538.30	-
			CB	35,127.94	35,589.97	1.30%	21,729.64	431
			CB+H	34,891.62	35,803.53	2.55%	21,666.42	369
			CB+CC	34,057.49	36,239.22	6.02%	21,616.17	178
6	1000	81	CB+H+CC	33,562.71	36,241.16	7.39%	21,654.41	197
			MB	35,295.29	35,295.29	0.00%	971.27	341
			MB+H	35,295.29	35,295.29	0.00%	1,100.93	326
			MB+CC	35,295.29	35,295.29	0.00%	9,223.33	319
			MB+H+CC	35,295.29	35,295.29	0.00%	11,142.54	389

Table 3.9 – Instance with 6 products and 81 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	33,280.16	36,576.38	9.01%	21,609.53	-
			CB	33,115.09	36,105.12	8.28%	21,765.39	190
			CB+H	33,538.78	35,790.16	6.29%	21,647.58	187
			CB+CC	26,682.87	38,022.24	29.82%	24,217.46	35
6	0	256	CB+H+CC	28,083.33	37,736.18	25.58%	21,944.72	40
			MB	34,451.74	34,451.74	0.00%	3,842.05	285
			MB+H	34,451.74	34,451.74	0.00%	6,262.13	410
			MB+CC	34,451.74	39,907.50	13.67%	21,895.63	65
			MB+H+CC	34,429.84	38,287.25	10.07%	21,838.79	70
			Monolithic	35,060.02	36,854.03	4.87%	21,608.35	-
			CB	33,600.90	36,892.35	8.92%	21,698.79	186
			CB+H	32,903.77	37,035.98	11.16%	32,325.77	188
			CB+CC	29,471.37	38,839.82	24.12%	*	30
6	1000	256	CB+H+CC	28,506.75	38,688.55	26.32%	25,079.78	45
			MB	35,060.02	35,060.02	0.00%	4,733.58	347
			MB+H	35,060.02	35,060.02	0.00%	7,202.22	448
			MB+CC	35,060.02	39,913.96	12.16%	22,077.57	80
			MB+H+CC	27,491.61	46,625.42	41.04%	21,600.00	5

* Execution reached the total amount of memory allowed.

Table 3.10 – Instance with 6 products and 256 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	-	-	-	21,621.15	-
			CB	33,036.91	35,923.02	8.03%	21,876.07	113
			CB+H	32,945.15	35,999.78	8.49%	21,731.70	101
			CB+CC	26,053.69	42,374.80	38.52%	*	5
6	0	625	CB+H+CC	26,053.69	43,160.81	39.64%	*	5
			MB	34,193.97	34,193.97	0.00%	16,152.40	360
			MB+H	34,193.97	34,193.97	0.00%	17,837.94	291
			MB+CC	26,053.69	43,247.82	39.76%	*	5
			MB+H+CC	33,922.85	44,814.07	24.30%	*	13
			Monolithic	-	-	-	21,621.67	-
			CB	32,497.52	37,015.75	12.21%	21,837.17	114
			CB+H	31,908.47	37,270.94	14.39%	21,858.08	98
			CB+CC	24,549.19	43,023.66	42.94%	*	5
6	1000	625	CB+H+CC	24,549.19	44,332.19	44.62%	*	5
			MB	34,880.38	34,880.38	0.00%	16,724.76	356
			MB+H	34,880.38	34,880.38	0.00%	19,797.74	314
			MB+CC	24,549.19	44,380.23	44.68%	*	5
			MB+H+CC	27,491.61	46,625.42	41.04%	*	5

* Execution reached the total amount of memory allowed.

Table 3.11 – Instance with 6 products and 625 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	55,439.17	58,442.98	5.14%	21,605.66	-
			CB	50,484.34	64,681.70	21.95%	21,666.29	442
			CB+H	51,102.18	63,831.85	19.94%	21,772.82	507
			CB+CC	46,632.97	63,709.48	26.80%	22,516.73	130
12	0	81	CB+H+CC	44,675.22	63,571.25	29.72%	22,205.16	150
			MB	55,433.48	61,017.34	9.15%	21,608.65	2,115
			MB+H	55,501.20	61,424.72	9.64%	21,604.37	1,735
			MB+CC	55,419.77	63,297.14	12.45%	21,652.34	315
			MB+H+CC	55,439.17	62,056.83	10.66%	21,636.85	320
			Monolithic	56,169.77	59,169.07	5.07%	21,605.96	-
			CB	52,134.08	65,299.08	20.16%	21,696.03	458
			CB+H	51,734.74	65,187.02	20.64%	21,665.77	447
			CB+CC	47,801.51	64,095.92	25.42%	21,934.02	105
12	1000	81	CB+H+CC	45,910.74	64,088.12	28.36%	21,877.99	145
			MB	56,169.77	61,325.38	8.41%	21,604.89	1,930
			MB+H	56,249.42	62,815.97	10.45%	21,603.88	1,783
			MB+CC	56,169.77	64,297.71	12.64%	21,604.13	305
			MB+H+CC	56,175.02	62,743.29	10.47%	22,286.51	245

Table 3.12 – Instance with 12 products and 81 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	-	-	-	21,615.41	-
			CB	36,621.68	59,548.02	38.50%	21,810.67	204
			CB+H	35,477.93	60,708.37	41.56%	21,769.32	171
			CB+CC	34,629.32	63,396.40	45.38%	*	5
12	0	256	CB+H+CC	34,629.32	55,975.29	38.13%	*	15
			MB	45,737.09	55,497.62	17.59%	21,610.27	670
			MB+H	45,756.34	56,820.73	19.47%	21,621.66	496
			MB+CC	34,629.32	63,927.58	45.83%	*	10
			MB+H+CC	44,963.00	63,587.85	29.29%	23,496.69	20
			Monolithic	-	-	-	21,617.57	-
			CB	39,200.11	59,888.91	34.55%	21,613.60	224
			CB+H	36,880.65	61,064.91	39.60%	21,909.67	171
			CB+CC	30,701.42	57,631.58	46.73%	*	5
12	1000	256	CB+H+CC	30,701.42	57,296.91	46.42%	23,881.59	15
			MB	46,273.26	60,055.04	22.95%	21,678.06	527
			MB+H	46,293.80	58,918.95	21.43%	21,614.10	452
			MB+CC	45,706.40	65,412.08	30.13%	*	29
			MB+H+CC	30,701.42	65,371.93	53.04%	*	10

* Execution reached the total amount of memory allowed.

Table 3.13 – Instance with 12 products and 256 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	-	-	-	21,641.11	-
			CB	44,141.80	80,824.70	45.39%	21,931.61	121
			CB+H	42,989.67	81,343.90	47.15%	21,923.29	127
			CB+CC	42,540.54	83,548.43	49.08%	*	5
12	0	625	CB+H+CC	42,540.54	84,250.43	49.51%	*	5
			MB	64,207.17	81,376.87	21.10%	21,678.92	260
			MB+H	64,129.09	76,647.69	16.33%	21,677.19	272
			MB+CC	42,540.54	84,494.45	49.65%	*	5
			MB+H+CC	42,540.54	84,264.90	49.52%	*	5
			Monolithic	-	-	-	21,645.42	-
			CB	45,328.80	83,039.43	45.41%	21,744.54	115
			CB+H	44,680.32	83,737.90	46.64%	21,895.01	131
			CB+CC	39,760.23	85,350.90	53.42%	*	5
12	1000	625	CB+H+CC	39,668.25	86,634.98	54.21%	*	5
			MB	64,848.43	80,205.05	19.15%	21,690.88	252
			MB+H	64,885.77	82,287.11	21.15%	21,771.56	239
			MB+CC	39,760.23	86,512.37	54.04%	*	5
			MB+H+CC	39,668.25	88,640.02	55.25%	*	5

* Execution reached the total amount of memory allowed.

Table 3.14 – Instance with 12 products and 625 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	90,341.77	99,507.39	9.21%	21,613.16	-
			CB	67,105.07	128,833.00	47.91%	21,607.23	269
			CB+H	67,105.07	129,728.31	48.27%	21,673.73	255
			CB+CC	67,105.07	114,071.54	41.17%	21,626.39	38
24	0	81	CB+H+CC	67,105.07	111,488.86	39.81%	22,208.15	60
			MB	90,709.59	112,509.59	19.38%	21,608.58	1,080
			MB+H	90,728.56	112,697.81	19.49%	21,609.70	1,132
			MB+CC	88,903.13	113,039.05	21.35%	22,014.27	60
			MB+H+CC	89,059.80	132,040.08	32.55%	24,147.66	61
			Monolithic	89,800.49	100,486.77	10.63%	21,609.45	-
			CB	62,027.09	131,093.64	52.68%	21,677.83	221
			CB+H	62,027.09	131,018.59	52.66%	21,612.24	230
			CB+CC	62,027.09	115,202.26	46.16%	23,289.52	40
24	1000	81	CB+H+CC	62,027.09	111,807.41	44.52%	21,621.70	55
			MB	91,150.52	117,726.59	22.57%	21,619.00	1,153
			MB+H	90,994.12	115,267.05	21.06%	21,624.06	1,067
			MB+CC	89,397.69	113,507.63	21.24%	22,085.62	75
			MB+H+CC	89,553.10	134,107.81	33.22%	*	55

* Execution reached the total amount of memory allowed.

Table 3.15 – Instance with 24 products and 81 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	-	-	-	21,632.83	-
			CB	69,149.34	131,738.64	47.51%	21,743.15	135
			CB+H	69,149.34	132,855.58	47.95%	22,268.11	145
			CB+CC	69,149.34	133,987.87	48.39%	*	5
24	0	256	CB+H+CC	69,149.34	134,804.56	48.70%	*	5
			MB	95,675.55	125,488.51	23.76%	21,663.49	219
			MB+H	95,642.66	120,637.20	20.72%	21,750.99	150
			MB+CC	69,149.34	135,562.01	48.99%	*	5
			MB+H+CC	69,149.34	136,239.94	49.24%	*	5
			Monolithic	-	-	-	21,676.96	-
			CB	64,518.52	135,794.92	52.49%	21,870.41	152
			CB+H	64,518.52	138,001.51	53.25%	21,841.07	136
			CB+CC	64,518.52	138,035.11	53.26%	*	5
24	1000	256	CB+H+CC	64,518.52	139,917.62	53.89%	*	5
			MB	96,001.70	126,951.50	24.38%	21,923.17	110
			MB+H	95,912.45	131,891.60	27.28%	21,899.53	115
			MB+CC	64,518.52	139,306.45	53.69%	*	5
			MB+H+CC	64,518.52	139,970.70	53.91%	*	5

* Execution reached the total amount of memory allowed.

Table 3.16 – Instance with 24 products and 256 scenarios

Products	γ	Scenarios		Lower Bound	Upper Bound	Gap	Time	# Iterations
			Monolithic	-	-	-	21,750.91	-
			CB	68,383.36	136,187.58	49.79%	22,219.27	73
			CB+H	68,383.36	136,928.79	50.06%	21,968.92	89
			CB+CC	68,383.36	138,002.49	50.45%	*	5
24	0	625	CB+H+CC	68,383.36	138,534.54	50.64%	*	5
			MB	97,325.23	134,981.14	27.90%	22,157.37	104
			MB+H	97,776.63	128,259.24	23.77%	21,760.02	99
			MB+CC	68,383.36	138,918.40	50.77%	*	5
			MB+H+CC	68,383.36	138,628.79	50.67%	*	5
			Monolithic	-	-	-	21,700.43	-
			CB	63,012.39	137,238.47	54.09%	21,614.62	77
			CB+H	63,012.39	138,335.81	54.45%	21,857.03	66
			CB+CC	63,012.39	139,274.51	54.76%	*	5
24	1000	625	CB+H+CC	63,012.39	139,773.61	54.92%	*	5
			MB	98,215.09	135,906.19	27.73%	21,842.04	94
			MB+H	97,459.075	136,016.63	28.35%	21,930.87	83
			MB+CC	63,012.39	139,974.67	54.98%	*	5
			MB+H+CC	63,012.39	139,949.60	54.97%	*	5

* Execution reached the total amount of memory allowed.

Table 3.17 – Instance with 24 products and 625 scenarios

Chapter 4

Integrating lot-sizing and scheduling under demand uncertainty

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An empirical study of the general lot-sizing and scheduling model under demand uncertainty via robust and stochastic approaches

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Abstract This paper presents an empirical assessment of the General Lot-Sizing and Scheduling Problem (GLSP) under demand uncertainty by means of a budget-uncertainty set robust optimization and a two-stage stochastic programming with recourse model. We have also developed a systematic procedure based on Monte Carlo simulation to compare both models in terms of protection against uncertainty and computational tractability. The extensive computational experiments cover different instances characteristics, a considerable number of combinations between budgets of uncertainty and variability levels for the robust optimization model, as well as an increasing number of scenarios and probability distribution functions for the stochastic programming model. Furthermore, we have devised some guidelines for decision-makers to evaluate a *priori* the most suitable uncertainty modeling approach according to their preferences.

Keywords Lot-sizing and Scheduling Problems; GLSP; Robust Optimization; Stochastic Programming; Empirical Study; Monte Carlo simulation

4.1. Introduction

There is a significant body of research developing mathematical models and efficient solution methods for combining lot-sizing and scheduling problems that reflects numerous real-world industrial applications whose corresponding optimization problems are computationally challenging (Almada-Lobo et al., 2015; Copil et al., 2016), e.g., glass container

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production planning (Fachini et al., 2016; Toledo et al., 2016), food and animal-feed industry (Claassen et al., 2016; Toso et al., 2009b), furnace scheduling (de Araujo et al., 2008), soft drink production (Ferreira et al., 2010), textile and fiberglass industries (Beraldi et al., 2008; Camargo et al., 2014), amongst many others.

The integration of lot-sizing and scheduling has received a great deal of attention as it promises to better use the expensive production resources by managing capacity and final inventories efficiently (Maravelias and Sung, 2009). In particular, the well-known General Lot-sizing and Scheduling Problem (GLSP) arises as the most popular small bucket product-oriented formulation for merging these decisions (Guimarães et al., 2014). The benefit of integrating decisions might be even more evident under uncertainty. However, relatively less attention has been devoted to understanding the role of uncertainty in the lot-sizing and scheduling problem.

In effect, many practical production planning problems are subjected to both environmental and system uncertainty (Ho, 1989; Mula et al., 2006), but most of the efforts to handle this problem assumes deterministic settings to find both optimal lot-sizes and production sequences (Guimarães et al., 2014). In practice, this means that the inventory levels, which are implicitly found in these decision models, are not strictly used for demand fulfillment strategies due to demand uncertainty. Therefore, a stochastic inventory management model is usually used afterwards to take into account overall uncertainties. The importance of studying models that acknowledge the stochastic nature of operations and integrate lot-sizing issues within scheduling in finite planning horizons has already been pointed out by Zhu and Wilhelm (2006). However, even over the past ten years, there have only been a few contributions focused on two- and multi-stage stochastic programs for the lot-sizing and scheduling problem.

Early attempts at modeling lot-scheduling problems without setups in a multi-stage framework can be found in Beraldi et al. (2006); Wu and Ierapetritou (2007). In a different direction, Ramezanian and Saidi-Mehrabad (2013) proposed a multi-level lot-sizing and scheduling problem with sequence-dependent setup times, stochastic demands and processing times, by means of chance-constrained programming. The authors assumed beforehand the probability distributions of both stochastic parameters to provide a tractable formulation. More recently, Hu and Hu (2016) tackled a single-level version of the same problem via two-stage stochastic programming. They presented a numerical study based on a small number of scenarios generated by moment matching and scenario reduction techniques. Other studies have assumed an infinite planning horizon for stochastic economic lot-scheduling problems (SELSP). The literature on the SELSP is reviewed in Sox et al. (1999) and Winands et al. (2011). Motivated by several practical process industries that have limited degrees of freedom when changing over between product families, Liberopoulos et al. (2013) analyzed the case of a SELSP with constraints in the production sequence by modeling this problem as a Markov decision process. Löhndorf et al. (2014) relaxed these assumptions on the changeovers and solved this problem within a simulation-optimization framework.

Using stochastic programming approaches to hedge against uncertainty, though, is sometimes criticized for being computationally prohibitive for combinatorial problems, especially for a large number of scenarios. At the same time, designing a plausible set of scenarios is often difficult due to the lack of historical data and/or to the excessive theoretical requirements for using the available scenario generation methods. The aforementioned drawbacks might be overcome by using robust optimization (RO) approaches, whose main goal rely on yielding "less sensitive" solutions to data variation, e.g., near-optimal solutions with a lower probability of being infeasible. Such a goal is usually achieved by modeling data uncertainty within bounded intervals that describe the uncertainty without needing the full knowledge of the probability distributions. The resulting model is thus optimized from a worst-case perspective, which is partially controlled by the risk preference of the decision-maker. Despite these potential advantages, RO has never been used to solve a lot-sizing and scheduling problem with sequence dependent setups. However, this approach has been successfully employed in more tactical and theoretical production planning problems. For example, Klabjan et al. (2013) proposed a robust minimax model for solving a single-item lot-size problem. The effectiveness of the model is validated by numerical experiments where it was proved that under certain conditions the robust model converges to the traditional stochastic model.

This paper thus contributes to this gap by developing a novel budget-uncertainty set robust optimization model for the GLSP under demand uncertainty. We have also developed a systematic procedure based on Monte Carlo simulation to compare our RO model to the traditional two-stage stochastic programming with recourse model in terms of protection against uncertainty and computational tractability. The extensive computational experiments cover different instances characteristics, a considerable number of combinations between budgets of uncertainty and variability levels for the robust optimization model, as well as an increasing number of scenarios and probability distribution functions for the stochastic programming model. Furthermore, we have devised some guidelines for decision-makers to evaluate a *priori* the most suitable uncertainty modeling approach according to their preferences. To the best of our knowledge, it is the first paper that proposes to investigate GLSP via robust optimization, also pointing out benefits and disadvantages of RO and stochastic programming. In fact, Sahinidis (2004) reiterated the need of a systematic comparison between the different uncertainty modeling approaches and Mula et al. (2006) also acknowledged that there was a need to compare different modeling approaches for production planning under uncertainty.

The remaining paper is organized as follows. Section 4.2 develops a robust optimization model with demand uncertainty for GLSP. Section 4.3 presents the stochastic programming model and the Monte Carlo experiment used to compare and assess the RO model. In Section 4.4 the computational experiment is described and its results are presented and commented. Finally, section 4.5 concludes this work and provides research directions.

4.2. General lot-sizing and scheduling models under uncertainty

The GLSP determines the production lot-sizes to fulfill demands and defines the best production sequence to minimize overall costs, assuming that setup is sequence-dependent. Mathematically, let J be the set of products indexed by j and ℓ, and T be the set of periods indexed by t. Assume that N_t is the subset of micro-periods of period t, such that

$\bigcup_{t\in T} N_t = N$. The deterministic GLSP can be posed as follows (Fleischmann and Meyr, 1997; Meyr, 2002):

(F1: `DetModel`)

$$\min \sum_{j\in J}\sum_{t\in T}\left(h_j^+\cdot I_{jt}^+ + h_j^-\cdot I_{jt}^-\right) + \sum_{(j,\ell)\in J}\sum_{n\in N} s_{j\ell}\cdot Z_{j\ell n} \tag{4.1}$$

$$\text{s.t.: } I_{j(t-1)}^+ + I_{jt}^- + \sum_{n\in N_t} X_{jn} = I_{jt}^+ + I_{j(t-1)}^- + d_{jt},\ \forall j\in J\wedge t\in T \tag{4.2}$$

$$\sum_{j\in J}\sum_{n\in N_t} p_j\cdot X_{jn} + \sum_{(j,\ell)\in J}\sum_{n\in N_t} q_{j\ell}\cdot Z_{j\ell n} \le cap_t,\ \forall t\in T \tag{4.3}$$

$$X_{jn}\le b_{jt}\cdot Y_{jn},\ \forall j\in J\wedge t\in T\wedge n\in N_t \tag{4.4}$$

$$\sum_{j\in J} Y_{jn} = 1,\ \forall n\in N \tag{4.5}$$

$$\sum_{\ell\in J} Z_{j\ell n} = Y_{j(n-1)},\ \forall j\in J\wedge n\in N \tag{4.6}$$

$$\sum_{j\in J} Z_{j\ell n} = Y_{\ell n},\ \forall \ell\in J\wedge n\in N \tag{4.7}$$

$$X_{jn}\ge m_j\cdot\left(Y_{jn} - Y_{j(n-1)}\right),\ \forall j\in J\wedge n\in N \tag{4.8}$$

$$Y_{jn}\in\mathbb{B},\ X_{jn},\ Z_{j\ell n},\ I_{jt}^+,\ I_{jt}^-\ge 0,\ \forall (j,\ell)\in J\wedge n\in N\wedge t\in T. \tag{4.9}$$

The input data h_j^+, h_j^-, $s_{j\ell}$, d_{jt}, p_j, $q_{j\ell}$, cap_t and m_j, represent holding cost, shortage cost, setup cost, demand, production time, setup time, capacity and minimum lot, respectively. Decision variables are related to production X_{jt}, inventory I_{jt}^+, backlogging I_{jt}^-, setup Y_{jn} and changeover between two products $Z_{j\ell n}$. The objective function (4.1) minimizes the overall costs. Constraints (4.2) and (4.3) refer to demand balance and capacity, respectively. Constraints (4.4) and (4.5) express the setup constraints and the requirement that only one setup state is defined in each micro-period, respectively. Parameter b_{jt} imposes an upper bound on the production amount. Constraints (4.6) and (4.7) state the relation between setup and changeover states. Constraints (4.8) enforce the minimum lot size if family j was not produced in the previous micro-period. A minimum lot size is needed as setup times do not always satisfy the triangular inequality (Fleischmann and Meyr, 1997; Toso et al., 2009a). Finally, constraints (4.9) define the domain of the variables.

4.2.1 Budget-uncertainty set robust optimization GLSP model

Following the uncertainty budget-uncertainty set robust optimization approach, we assume that uncertain demands belong to an "uncertainty space", which is modeled via a convex polyhedral set composed by the corresponding nominal (deterministic) value of the parameter as well as a deviation from the nominal value. Polyhedral sets provide tractable robust counterparts, which is particularly appealing to deal with combinatorial problems under uncertainty. Furthermore, it is possible to control the conservativeness of the robust solution by limiting the number of parameters that achieve their worst-case values.

To determine the robust counterpart of the GLSP with demand uncertainty, we first rewrite the balancing constraints (4.2) as a set of inequalities. Otherwise, those constraints might be trivially infeasible depending on data variation; see a more detailed discussion in Gorissen et al. (2015). For this purpose, let us define the net inventory for product j in period t by $I_{jt} = I^+_{jt} - I^-_{jt}$, with I_{jt} being unrestricted in sign. Clearly, $I_{jt} = I_{j0} + \sum_{\tau=1}^{t}(\sum_{n\in N_\tau} X_{jn} - d_{j\tau})$, which yields the following two balancing constraints:

$$H_{jt} \geq h^+_{jt} \cdot I_{jt} = h^+_{jt} \cdot \left[I^+_{j0} + \sum_{\tau=1}^{t} \sum_{n\in N_\tau} X_{jn} - \sum_{\tau=1}^{t} d_{j\tau} \right], \ \forall j \in J \wedge t \in T, \tag{4.10}$$

and

$$H_{jt} \geq h^-_{jt} \cdot (-I_{jt}) = h^-_{jt} \cdot \left[I^-_{j0} - \sum_{\tau=1}^{t} \sum_{n\in N_\tau} X_{jn} + \sum_{\tau=1}^{t} d_{j\tau} \right], \ \forall j \in J \wedge t \in T. \tag{4.11}$$

H_{jt} denotes the holding costs or shortage costs of product j in period t according to the sign I_{jt} takes. Now, following the robust optimization perspective advocated by Bertsimas and Sim (2004) and Bertsimas and Thiele (2006), the uncertainty demand in both constraints (4.10) and (4.11) is modeled over the polyhedral-uncertainty set U:

$$U = \left\{ \mathbf{D} \in \mathbb{R}_+^{|J|\times|T|} |\ \tilde{d}_{jt} \in [d_{jt} - \hat{d}_{jt}, d_{jt} + \hat{d}_{jt}],\ \sum_{\tau=1}^{t} \frac{|\tilde{d}_{j\tau} - d_{j\tau}|}{\hat{d}_{j\tau}} \leq \Gamma_{jt},\ \forall (j \in J \wedge t \in T) \right\}. \tag{4.12}$$

in which $\mathbf{D} = [\tilde{d}_{ij}]$ indicates that all the $|J| \times |T|$ coefficients are subject to uncertainty. However, it would be possible to reduce the dimension of matrix $\mathbf{D}$ by modeling only a subset of uncertain coefficients, say, $\mathbf{D}' \subseteq \mathbf{D}$. Notice that the uncertainty set considers that the cumulative random variable $\tilde{d}_{j\tau}$ is bounded and symmetrically distributed around the half-length of the interval $\left[d_{j\tau} - \hat{d}_{j\tau},\ d_{j\tau} + \hat{d}_{j\tau}\right]$. It is also usual to define a scale deviation $\xi_{jt} = \left(\tilde{d}_{jt} - d_{jt}\right)/\hat{d}_{jt}$ that belongs to the interval $[-1, 1]$, such that $\tilde{d}_{jt} = d_{jt} + \hat{d}_{jt}\xi_{jt}$.

The *budget of uncertainty* parameter Γ_{jt} limits the allowed realizations of data within the range around the nominal values, as shown in (4.12). Basically, Γ_{jt} controls the size of the uncertainty set or equivalently the maximum number of coefficients that can assume their worst-case value for each product j and period t. The budget can also reflect risk preferences. Risk free managers are insensitive to risk and/or random events, so they can assign $\Gamma_{jt} = 0$ to optimize the nominal (deterministic) problem only. Very conservative risk averse managers might adopt larger budgets to be protected for any realization of the random variables within the uncertainty set, i.e, $\Gamma_{jt} = t$. Finally, $\Gamma_{jt} \in [0, t]$ enables risk averse managers to trade-off robustness and cost to construct a myriad of feasible (maybe optimal) solutions for the actual problem. Typically, we assume that the uncertainty increases with the number of time periods and that it does not make sense to increase the budget greater than the actual increase in the periods, the formulation also holds $\Gamma^d_{j1} \leq \Gamma^d_{j2} \leq \cdots \Gamma^d_{j|\mathcal{T}|}$ and $\Gamma_{jt} - \Gamma^d_{j(t-1)} \leq 1,\ \forall (j,t)$ (Bertsimas and Thiele, 2006; Bienstock and Ozbay, 2008; Alem and Morabito, 2012).

The worst-case realization of the demand uncertainty is finally achieved by solving the nonlinear new constraints (4.13) and (4.14) over the uncertainty set, i.e.:

$$H_{jt} \geq h^+_{jt} \cdot I_{jt} = h^+_{jt} \cdot \left[I^+_{0t} + \sum_{\tau=1}^{t} \sum_{n \in N_\tau} X_{jn} - \min_{\tilde{\mathbf{d}} \in U} \sum_{\tau=1}^{t} (d_{j\tau} + \hat{d}_{j\tau} \cdot \xi^d_{j\tau}) \right],\ \forall j \in J \wedge t \in T, \quad (4.13)$$

and

$$H_{jt} \geq h^-_{jt} \cdot (-I_{jt}) = h^-_{jt} \cdot \left[I^-_{0t} - \sum_{\tau=1}^{t} \sum_{n \in N_\tau} X_{jn} + \max_{\tilde{\mathbf{d}} \in U} \sum_{\tau=1}^{t} (d_{j\tau} + \hat{d}_{j\tau} \cdot \xi^d_{j\tau}) \right],\ \forall j \in J \wedge t \in T. \quad (4.14)$$

Both inner optimization problems in contraints (4.13) and (4.14) lead to the primal problem for each pair (j,t) depicted in (4.15), which is transformed to its dual in (4.16):

$$\begin{aligned} \max \quad & \sum_{\tau=1}^{t} \hat{d}_{j\tau} \cdot \xi_{j\tau} \\ \text{s.t.:} \quad & \sum_{\tau=1}^{t} \xi_{j\tau} \leq \Gamma_{jt}, \\ & 0 \leq \xi_{j\tau} \leq 1,\ \forall \tau \leq t. \end{aligned} \quad (4.15) \qquad \begin{aligned} \min \quad & \Gamma_{jt} \cdot \lambda^d_{jt} + \sum_{\tau=1}^{t} \mu^d_{j\tau t} \\ \text{s.t.:} \quad & \lambda^d_{jt} + \mu^d_{j\tau t} \geq \hat{d}_{j\tau},\ \forall \tau \leq t \\ & \lambda^d_{jt},\ \mu^d_{jt} \geq 0,\ \forall \tau \leq t. \end{aligned} \quad (4.16)$$

The decision variable ξ^d_{jt} can be seen as a binary decision that assumes 1 only if the maximum deviation of the parameter d_{jt} is taken into account. However, as the corresponding technological matrix of the subproblem (4.15) is totally unimodular, then it follows that ξ^d_{jt} can be modeled as a continuous variable in the interval $[0,1]$.

The robust linear counterpart of the GLSP with uncertainty demand is obtained thus by incorporating the dual of the auxiliary problem into formulations (4.13) and (4.14) in an attempt to produce a mathematical model as tractable as the original formulation:

(F2: `RobModel`)

$$\min \sum_{j \in J} \sum_{t \in T} H_{jt} + \sum_{(j,\ell) \in J} \sum_{n \in N} s_{j\ell} \cdot Z_{j\ell n} \quad (4.17)$$

s.t.: Constraints(4.3), (4.4), (4.5), (4.6), (4.7), (4.8)

$$H_{jt} \geq h^+_{jt} \cdot \left(I^+_{0t} + \sum_{\tau=1}^{t} \sum_{n \in N_\tau} X_{jn} - \sum_{\tau=1}^{t} d_{j\tau} + \Gamma_{jt} \cdot \lambda_{jt} + \sum_{\tau=1}^{t} \mu_{j\tau t} \right),\ \forall j \in J \wedge t \in T \quad (4.18)$$

$$H_{jt} \geq h^-_{jt} \cdot \left(I^-_{0t} - \sum_{\tau=1}^{t} \sum_{n \in N_\tau} X_{jn} + \sum_{\tau=1}^{t} d_{j\tau} + \Gamma_{jt} \cdot \lambda_{jt} + \sum_{\tau=1}^{t} \mu_{j\tau t} \right),\ \forall j \in J \wedge t \in T \quad (4.19)$$

$$\lambda_{jt} + \mu_{j\tau t} \geq \hat{d}_{j\tau}, \ \forall j \in J \wedge t \in T \wedge \tau \leq t \tag{4.20}$$

$$\lambda_{jt}, \ \mu_{j\tau t} \geq 0, \ , \ \forall j \in J \wedge t \in T \wedge \tau \leq t \tag{4.21}$$

$$Y_{jn} \in \mathbb{B}, \ X_{jn}, \ Z_{j\ell n} \geq 0, \ \forall (j,\ell) \in J \wedge n \in N \wedge t \in T. \tag{4.22}$$

The robust GLSP assumes that uncertainty affects the "cumulative" demands over the planning horizon in an attempt to avoid very pessimistic solutions. In fact, in the subproblem (4.15), notice that the budget of uncertainty Γ_{jt} can be "shared" among all the decision variables ξ up to the current period t, i.e., for $t = 2$, we have $\xi_{j1} + \xi_{j2} \leq \Gamma^d_{j2}$. If we assume that uncertainty relies on each $\tilde{d}_{jt}$, that would lead to the following constraints $\xi_{j1} \leq \Gamma^d_{j1}$ and $\xi_{j2} \leq \Gamma^d_{j2}$, thus having only $\Gamma_{jt} = 0$ or 1, which means either nominal or worst-case scenario. Finally, notice that this model naturally ensures $H_{jt} \geq 0$, as the right-hand sides of the reformulated balancing constraints are always non-negative.

Let $X^{\star}_{jn}$ be the optimal solution of the robust optimization model. Then, the probability of violating the demand balancing constraints can be evaluated as follows (Wei et al., 2010):

$$Pr\left[H_{jt} < h^{+}_{jt} \cdot \left(\sum_{\tau=1}^{t} \sum_{n \in N_t} X^{\star}_{jn} - \sum_{\tau=1}^{t} \tilde{d}_{j\tau}\right)\right] \approx 1 - \Phi\left(\frac{\Gamma_{jt} - 1}{\sqrt{t}}\right) \tag{4.23}$$

and

$$Pr\left[H_{jt} < h^{-}_{jt} \cdot \left(-\sum_{\tau=1}^{t} \sum_{n \in N_t} X^{\star}_{jn} + \sum_{\tau=1}^{t} \tilde{d}_{j\tau}\right)\right] \approx 1 - \Phi\left(\frac{\Gamma_{jt} - 1}{\sqrt{t}}\right) \tag{4.24}$$

where $\Phi(\theta)$ is the cumulative distribution function of a standard normal for all pairs (j, t). These bounds on the probability of constraints violation might be used to define a *priori* a reasonable budget of uncertainty, thus avoiding to solve the robust counterpart several times for each budget of uncertainty. For example, if the decision maker would like to guarantee that a given threshold (cost or profit) will not be violated more than a confidence level α, we have that $\Gamma_{jt} = 1 + \Phi^{-1}(1 - \alpha) \cdot \sqrt{t}$.

4.2.2 Two-stage stochastic programming GLSP model

A natural benchmark for assessing the performance of RO models is solving the corresponding two-stage model. In traditional scenario-based two-stage stochastic approaches, uncertainty is handled via a finite set of outcomes or scenarios $k \in K$ in some probability space. Scenarios may represent the realizations of the random variables. For this modeling, we commonly assign a probability π_k for the occurrence of scenario k, such that $\pi_k > 0$ and $\sum_k \pi_k = 1$. In this paper, we consider independent realizations for demands.

The flexibility of reacting to the uncertainty outcomes is linked to several factors, such as the production technology/capital insensitivity, the planning horizon, and the planner attitude towards risk. Here, the stochastic programming model considers that both production quantities and production sequences have to be defined in the first stage, i.e., before uncertainty unveils. In the second stage, the model reacts to the uncertainty outcomes by adjusting the demand fulfillment in a simple recourse formulation, as follows:

(F3: `StochModel`)

$$\min \sum_{j\in J}\sum_{t\in T}\sum_{k\in K}\pi_k \cdot \left(h_j^+ \cdot I_{jtk}^+ + h_j^- \cdot I_{jtk}^-\right) + \sum_{(j,\ell)\in J}\sum_{n\in N} s_{j\ell}\cdot Z_{j\ell n} \tag{4.25}$$

s.t.: Constraints (4.3), (4.4), (4.5), (4.6), (4.7), (4.8)

$$I_{j(t-1)k}^+ + I_{jtk}^- + \sum_{n\in N_t} X_{jn} = I_{jtk}^+ + I_{j(t-1)k}^- + d_{jtk},\ \forall j\in J \wedge t\in T \wedge k\in K \tag{4.26}$$

$$Y_{jn}\in\mathbb{B},\ X_{jn},\ Z_{j\ell n},\ I_{jtk}^+,\ I_{jtk}^- \geq 0,\ \forall (j,\ell)\in J \wedge n\in N \wedge t\in T \wedge k\in K. \tag{4.27}$$

Differently from the RO model, the objective function reflects an expected cost. Constraints (4.26) must be feasible for all scenarios $k \in K$. The main goal of the two-stage program is to find a good compromise solution for both production quantities and sequences so as to minimize the excess of inventory and backorder costs incurred in the second-stage for each scenario. This model considers fixed production planning and scheduling for the entire time horizon, which can be applied to conservative planning and in rigid production environments, such as the steel production planning (Mattik et al., 2014).

4.3. Solution approach: A Monte Carlo sampling procedure

The benefits and drawbacks of both GLPS models under uncertainty are empirically assessed via a Monte Carlo sampling procedure. Figure 4.1 summarizes the proposed Monte Carlo procedure based on Wang (2008). Firstly, deterministic, robust, and stochastic models are solved by a commercial solver until the (sub) optimal solution is found. Then, production and setup variables are fixed in their primal values and random realizations for demands are generated from distinct probability density functions (PDF) in order to analyze the effect of having made the right or wrong assumption on the true underlying distribution. After a considerable number of realizations it is possible to compare average costs, standard deviations, and worst- and best-case scenario costs of all approaches. For comparison purposes, this paper proposes to fix the corresponding here-and-now decisions of the two-stage model in all models (F1, F2 and F3). Then all the proposed models are solved with only the wait-and-see decisions free and the corresponding average cost for each demand realization is obtained.

Three different PDFs are analyzed in the experiment: uniform, gamma, and log-normal. Each demand distribution has the mean and coefficient of variation values previously determined for the respective product and period. The variability of the realizations is based on the coefficient of variation given by the ratio between standard deviation and mean value. In this study, the values chosen were 0.2 and 0.5. The simulation procedure was repeated 1,000 times for each demand distribution to achieve statically significant results. The same seed generator was used for all simulations.

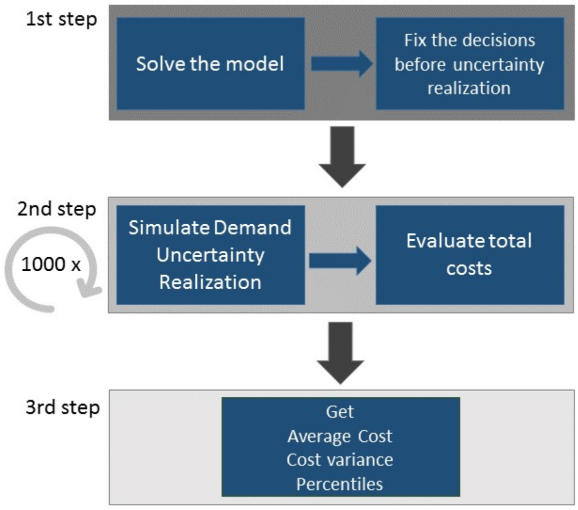


Figure 4.1 – Overall framework of the Monte Carlo procedure.

The experiment for the robust model was performed with the variability level $\hat{\gamma}$ varying from 0 to 3 in steps of 0.1, where $\hat{d}_{j\tau} = \hat{\gamma} \cdot \sigma$ and σ is the known demand standard deviation. We have also tested 4 different values for the budget of uncertainty parameter Γ_{jt}: t, $0.5t+0.5$, $0.1t+0.5$ and $0.05t+0.1$, where t is the time period. These budgets of uncertainty represent different attitudes towards risk. $\Gamma_{jt} = t$ represents the worst-case or Soyster approach, where all the uncertain coefficients are assumed to vary in their worst-case values for all j and t. Naturally, it is not necessary to use robust optimization to protect against this extreme case. In fact, it suffices to replace nominal demands by $d_{jt} + \hat{d}_{jt}$. For this reason, we have analyzed less pessimistic budgets, as already studied in recent literature (Adida and Perakis, 2010; Alem and Morabito, 2012). Figure 4.2 shows the cumulative protection over the time horizon for each proposed budget of uncertainty. For example, when $\Gamma_{jt} = 0.5t+0.5$ we have that the minimum protection is 1 and the maximum is 3.0 when $t = 5$. This means that the first period is fully protected, but the last period is protected against the variation of 60% of the coefficients.

We have tested 5 different sizes of scenario-trees for the two-stage stochastic programming model: 100, 200, 300, 400, and 500. The scenarios were randomly generated using the mean and the coefficient of variation of product demands in each time period t, which are assumed to have a distribution of probability given by uniform, gamma, or log-normal. All the assumed distributions are combined with the true underlying distribution obtained from the Monte Carlo simulation in order to analyze how much an inaccurate distribution impacts on the average cost. Figure 4.3 shows the overall scheme and the combinations of parameters performed for both robust and stochastic models. This numerical study help us to evaluate the combination of parameters that has the best performance in terms of average cost and/or worst-case value.

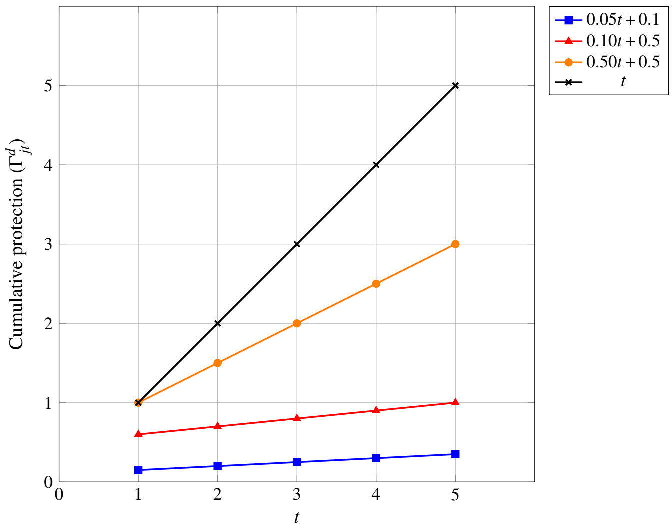


Figure 4.2 – Cumulative protection of the budgets of uncertainty considered over the time horizon.

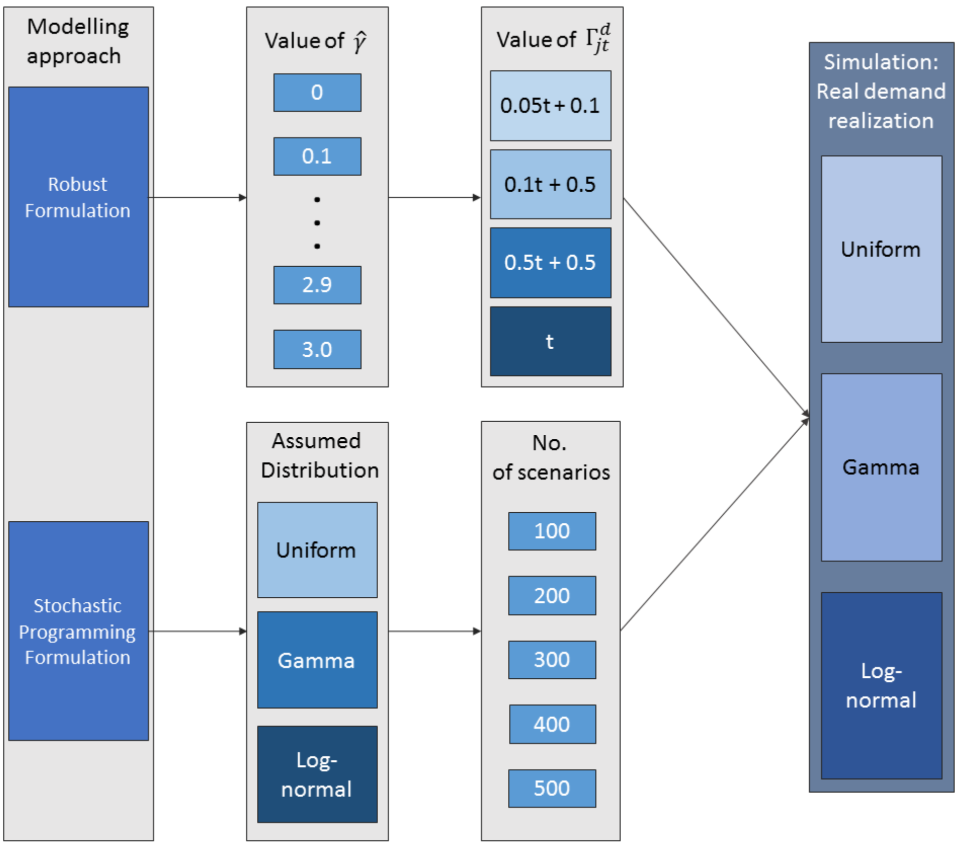


Figure 4.3 – Simulation framework for both robust and stochastic approaches.

4.4. Computational experiments

The goals of the computational tests are threefold: (i) Evaluate and compare the performance of both robust and stochastic approaches in terms of average costs, conservativeness, and computational efficiency. (ii) Find patterns in how each approach hedges against uncertainty on different instances characteristics. (iii) Provide a practical guideline on how to select the most suitable method based on the advantages and disadvantages of each one. This section is organized as follows. Subsection 4.4.1 shows the characteristics of the proposed instances. Subsection 4.4.2.1 presents the comparisons among the models using the proposed Monte Carlo procedure. Finally, subsection 4.4.2.3 discusses managerial insights into the usage of both approaches and provides some guidelines for decision makers. All the computational experiments were implemented in C++ language and the models were solved using IBM ILOG CPLEX Optimization Studio 12.4 on an Intel E5-2450 processor under a Scientific Linux 6.5 platform.

4.4.1 Instance generation

In order to analyze the impact of the input parameters on the simulation results, we generated 80 instances divided into 8 different classes. The instances present the same number of products, periods, and micro-periods: $|J| = 5$, $|T| = 4$ and $|N_t| = 5$, respectively. The mean value of demands (μ_{jt}) is randomly generated from a uniform distribution $U(120,480)$ in all instances. The capacity (cap_t) is derived from the mean demand. Other parameters, such as production time, minimum and maximum production lot are also equal for all instances: $p_j = 1$, $m_j = 1$ and $b_{jt} = cap_t$, respectively. The instances of the same class differ by setup time, setup costs and holding costs, which are random generated from the following uniform distribution: $q_{j=\ell} = U(1,10)$, $q_{j\neq\ell} = U(11,50)$, $s_{j=\ell} = 0$, $s_{j\neq\ell} = q_{j\ell} \cdot U(0,1)$ and $h_j^+ = U(1,10)$. The differences between each class of instances rely on the coefficient of variation (CV) of the demand, expected capacity utilization, and relation of shortage and holding costs, as shown Table 4.1.

Class	cap_t	h_j^-	CV
1	$\frac{\sum_j \mu_{jt}}{0.6}$	$2h_j^+$	0.2
2	$\frac{\sum_j \mu_{jt}}{0.6}$	$2h_j^+$	0.5
3	$\frac{\sum_j \mu_{jt}}{0.6}$	$10h_j^+$	0.2
4	$\frac{\sum_j \mu_{jt}}{0.6}$	$10h_j^+$	0.5
5	$\frac{\sum_j \mu_{jt}}{0.9}$	$2h_j^+$	0.2
6	$\frac{\sum_j \mu_{jt}}{0.9}$	$2h_j^+$	0.5
7	$\frac{\sum_j \mu_{jt}}{0.9}$	$10h_j^+$	0.2
8	$\frac{\sum_j \mu_{jt}}{0.9}$	$10h_j^+$	0.5

Table 4.1 – Characteristics of each class of instances

4.4.2 Results and discussions

4.4.2.1 Average and worst-case analysis

As already mentioned, we tested the RO model for 30 variability levels and 4 budgets of uncertainty, whereas the SP model was analyzed for 5 sizes of scenario-trees and 3 assumed PDFs. For each combination of robust and/or stochastic parameters and class of instances, we have 10 instances and 1,000 runs of the Monte Carlo sampling procedure, totaling 10,000 simulation runs. We then evaluate the average performance of both models for each combination of parameters over 10,000 simulation runs. For each class of instances and true underlying distribution, we also have the corresponding (best) combination of parameters that lead to the lowest average cost (Table 4.2) and and worst-case scenario cost (Table 4.3). The headings of both tables are as follows: the class of instances (column # 1), the true underlying demand distribution (column # 2), the objective value (columns # 3, 9, and 15), the average cost obtained from the Monte Carlo simulation (columns # 4, 10, and 16), the standard deviation of average cost (columns #5, 11, and 17), the average worst-case value (columns #6, 12, and 18), the average best-case value (columns # 7, 13, and 19), and the runtimes (columns # 8, 14, and 20). In addition, columns # 21, 22, and 23 show the relative differences on the average and worst-case costs for each two approaches. Further details on the combination of parameters that achieved the best performance for both stochastic and robust approaches is depicted in Tables 4.4–4.6 of the supplementary material.

Not surprisingly, the deterministic approach fails in hedging against uncertainty for all class of instances. Indeed, the main performance metrics reveals that under uncertainty its solutions are, on average, at least 31% more expensive, 39% less stable in terms of standard deviation, and 29% less robust in terms of worst-case cost, in comparison to the solutions provided by the remaining approaches. These figures are still more pronounced in classes 3 and 4 due to their high shortage costs. On the other hand, tighter capacity and larger coefficient of variation apparently reduce the cost difference amongst the approaches. Notice that average costs provided by the deterministic approach are always cheaper when the true underlying distribution is log-normal, probably because it is less likely to generate backlogging due to its skewness. Following a similar rationale, it is clear that average costs are always more expensive when the underlying distribution is uniform. Last, but not least, it is worth noting that, "if everything goes right", the deterministic approach yields a better best-case cost, as expected. Of course, when there is no disruption, one should favor the deterministic approach to save approximately 12%.

As expected, the stochastic model presents the best average performance over the proposed class of instances and simulation runs. On average, the costs provided by the stochastic model are 2.11% lower. In classes 7 and 8, though, the stochastic model has an average cost up to 4.5% lower than those provided by the robust optimization model, probably because tighter capacity and higher backlogging costs provoke a worsen increase in the objective values of the RO model as a result of the worst-case deviation of demands. However, it is clear from Table 4.3 that RO outperforms SP in terms of standard deviation with 23% less dispersed values and almost 12% lower worst-case costs, on average. In particular, class 3 reveals that it is possible to reduce by 68% the costs of the most pessimistic solutions and by 46% its corresponding standard deviation.

All the aforementioned phenomena are in accordance with both SP and RO paradigms, i.e., SP minimizes (expected) average costs while RO minimizes the maximum (worst) cost. Over the past years, many authors have confirmed such behavior for different problems, e.g., Adida and Perakis (2010); Thorsen and Yao (2015); Melamed et al. (2016). As a consequence, when the priority is to minimize the worst-case cost, the best performance will occur for more conservative budgets and higher variability levels, which in turn will increase the average cost. However, as the RO model tradeoffs performance and conservativeness, it is also possible to choose a good combination of budget and variability to obtain a good average cost. This might be achieved, e.g., by selecting less conservative budgets of uncertainty in an attempt to be partially protected against worst-case deviations, while not increasing dramatically the so-called "price of robustness" (Bertsimas and Sim, 2004). In fact, the results given by classes 1 and 2 show some examples in which RO outperforms SP in terms of both standard deviation and worst-case cost, still providing better average costs, even when the best combination of parameters is focusing on minimum average costs. This is achieved by selecting a less conservative budget ($\Gamma_{jt} = 0.10t + 0.5$) most times; see columns #10 and #16 in Table 4.2. When the focus is on minimum worst-case cost, we found many examples, mainly in the first three classes of instances, where solutions are more stable and worst-case costs are much lower at the expense of a minor deterioration in average costs.

Figure 4.4 tradeoffs average against worst-case costs in terms of the relative differences $\Delta^{avg}(\cdot)$ and $\Delta^{wc}(\cdot)$ to illustrate the previous results. The argument $(\cdot)$ refers to either the minimum average cost focus "avg" exhibited in Table 4.2 or the minimum worst-case cost focus "wc" exhibited in Table 4.3. Also, $\Delta^{avg}(\cdot) = \frac{z^{avg}_{RO} - z^{avg}_{SP}}{z^{avg}_{RO}}$ and $\Delta^{wc}(\cdot) = \frac{z^{wc}_{RO} - z^{wc}_{SP}}{z^{wc}_{RO}}$, in which z^{avg}_{RO} (z^{avg}_{RO}) refers to the average cost of the RO (SP) model and z^{wc}_{RO} (z^{wc}_{SP}) is the worst-case cost of the RO (SP) model. The closer to the left and bottom the solutions are, the better is the RO performance in comparison to the SP performance. Notice that the RO model is able to substantially mitigate worst-case costs at the expense of a more slowly increase in average costs, mainly when the best combination is focused on minimum worst-case costs, as already discussed. In these cases, it is preferable to select more conservative budgets to be protected against larger deviations of the demand uncertainty. These results indicate that the RO model also has a good performance regarding average costs, but the SP model rarely has good worst-case costs performance.

Apparently, the assumed demand distribution has a relatively small impact on the performance of the SP model in terms of average values for the proposed instances. The major difference between assumed and true underlying demand distribution is only 2.46% in terms of average cost. This might happen because we have an increased chance to sample similar scenarios amongst the three types of distributions as the number of realizations increases as well. And, in fact, in most cases the best average (worst-case) costs are attained with large number of 500 (200) scenarios. For this reason, while runtimes of both deterministic and robust models have the same order of magnitude, the stochastic model is approximately 30 times more expensive computationally, on average. In class 1, though,

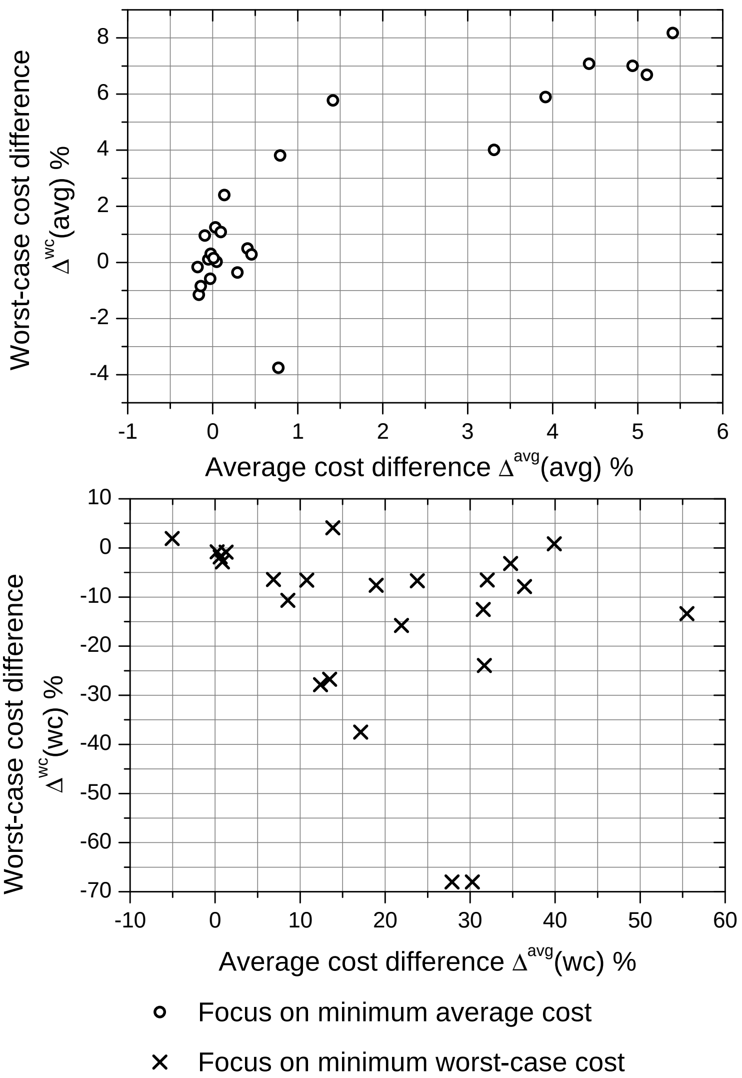
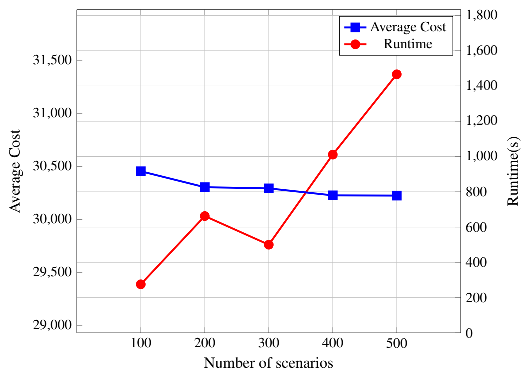


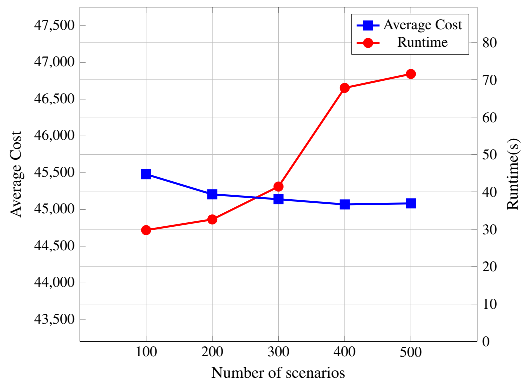
Figure 4.4 – Tradeoffs between average against worst-case costs. The above figure refers to the values that lead to the best combination of parameters to minimize average costs (Table 4.2). The below figure refers to the values that lead to the best combination of parameters to minimize worst-case costs (Table 4.3).

the SP model is hundreds of times slower than the RO model.

All these results are in accordance with the size of the mathematical models. In fact, as it is reasonable to assume that the number of scenarios is greater than the number of periods, then $|K| \gg |L|$ hold, where $L = \{\tau | \tau \leq t,\ t \in T\}$. Consequently, we can say that the SP model has $|J| \times |T| \times (|K| - |L| - 1)$ more constraints and $|J| \times |T| \times (2|K| - |L| - 1)$ more variables than the RO model. In our particular class of instances, we have 9,780 more constraints and 19,780 more variables with 500 scenarios than the RO model. Figure 4.5 shows a clear tradeoff between performance and tractability as the number of scenarios increases. The solution becomes more accurate, but in exchange of a substantial increase in runtimes. We also compared RO and SP approaches when both present the same magnitude of runtimes, which is true when the SP model has approximately 10 scenarios. In this case, the SP model has considerable worse performances than the robust optimization model in most classes of instances in terms of average and worst-case costs, and standard deviation. RO has also clear advantages over SP when short runtimes are necessary (see Table 4.7 in the supplementary on line material).



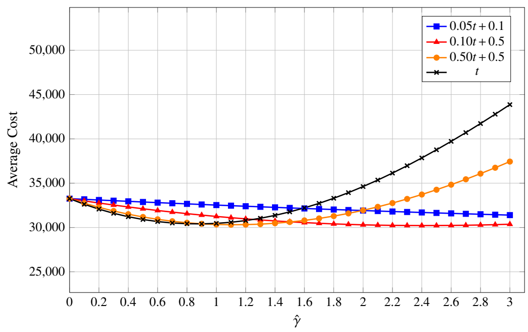
(a) Instance class 2 with uniform distribution.



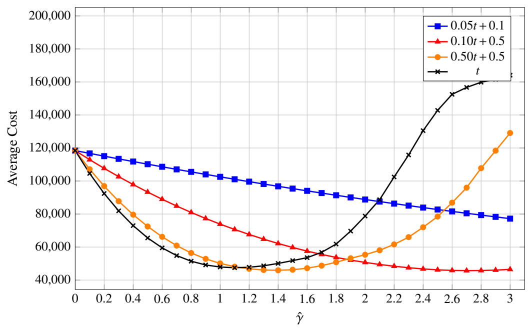
(b) Instance class 4 with uniform distribution.

Figure 4.5 – Stochastic programming performance according to the number of scenarios.

According to columns # 13 and 15 of Table 4.2, the objective value of the stochastic programming model is fairly precise on the simulation average cost, which can be considered an advantage to better “predict” the model’s performance. The robust objective value, though, overestimates the average cost for the first 5 instance classes and underestimates it for the last 3 instance classes. This happens because the robust optimization model is worst-case oriented, thus minimizing the cost for the worst-case demand deviation according to its variability level and budget of uncertainty, which differs from the average cost in the simulation experiment. The under or overestimate of the RO objective function is related to its budget of uncertainty and variability level (see Table 4.5): protective budget of uncertainty and/or high variability level lead to overestimation of average cost (first 5 instance classes) and less conservative budget of uncertainty and low variability level lead to underestimation of average cost (last 3 instance classes). Figure 4.6 show the performance of the robust and stochastic models according to variability level and number of scenarios. The combination of the parameters Γ_{jt} and $\hat{d}_{j\tau}$ has a critical impact on the robust optimization model performance. For the stochastic programming model, after a certain number of scenarios, it has a lower impact on the solution quality. It is also worth of note that the demand distribution does not have a great impact on the performance of the best combination of Γ_{jt} and $\hat{\gamma}$ for the robust optimization model. Hence, it suffices to decide on a single combination of parameters for each class of instances considering any demand distributions.



(a) Instance class 2 with uniform distribution.



(b) Instance class 4 with uniform distribution.

Figure 4.6 – Robust optimization model performance according to the different values of Γ_{jt} and $\hat{\gamma}$.

		Deterministic						Robust						Stochastic						Cost Difference		
Class #1	Distribution #2	Obj. Function #3	Average #4	Std Dev. #5	Worst-case #6	Best-case #7	Runtime(s) #8	Obj. Function #9	Average #10	Std Dev. #11	Worst-case #12	Best-case #13	Runtime(s) #14	Obj. Function #15	Average #16	Std Dev. #17	Worst-case #18	Best-case #19	Runtime(s) #20	Det.–Robust #21	Det.–Stoch #22	Robust–Stoch #23
1	Uniform	117	14917	4535	33684	5938	12	23270	13534	3382	25488	5455	12	13473	13541	3381	25461	5488	4033	9.27%	9.23%	−0.05%
	Gamma	117	14548	5511	40207	4314	12	19566	13498	4385	33775	4257	14	13377	13520	4418	34165	4190	3051	7.22%	7.06%	−0.16%
	Log-normal	117	14436	5419	36392	4349	12	17713	13588	4352	33117	4861	11	13386	13607	4369	33397	4748	5005	5.87%	5.74%	−0.14%
2	Uniform	131	33257	10040	74788	12946	12	50093	30218	7622	59064	12268	14	30000	30225	7604	58883	12066	1939	9.14%	9.11%	-0.02%
	Gamma	131	31991	12919	98631	7763	12	35520	30397	10668	89382	9535	19	30041	30425	10622	88525	9645	2278	4.98%	4.89%	−0.09%
	Log-normal	131	30807	13435	102458	7538	12	27746	29882	11516	95921	10409	11	29974	29935	11543	96082	9924	2550	3%	2.83%	−0.18%
3	Uniform	84	45216	23287	141329	7829	7	30871	17403	5012	43755	7683	9	17034	17265	4893	42089	7226	397	61.51%	61.82%	0.79%
	Gamma	84	44461	26413	167418	6619	7	30871	19317	8043	75304	7252	9	17034	19238	7981	74931	7128	397	56.55%	56.73%	0.41%
	Log-normal	84	43012	26042	146766	6188	7	30871	19599	8093	77208	7810	9	17034	19542	8171	77489	8072	397	54.43%	54.57%	0.29%
4	Uniform	108	118430	60681	374027	19806	9	76981	45715	14364	123259	17978	10	44596	45068	12941	116149	17497	79	61.4%	61.95%	1.42%
	Gamma	108	114599	72841	481681	12005	9	81911	54509	25396	260755	23617	37	44409	54088	27142	270554	21406	55	52.43%	52.8%	0.77%
	Log-normal	108	108565	74862	479698	13396	9	76981	56052	32406	302733	21890	10	44759	55796	31232	301876	21661	126	48.37%	48.61%	0.46%
5	Uniform	78	14289	4397	32626	5612	10	17250	13093	3445	25896	5372	15	12984	13075	3382	25275	5578	272	8.37%	8.5%	0.14%
	Gamma	78	13907	5308	40672	4175	10	17242	12930	4298	34953	3660	35	12755	12924	4318	34945	3688	2102	7.03%	7.07%	0.05%
	Log-normal	78	13698	5197	34454	4171	10	15264	12943	4259	31281	4342	15	12889	12939	4205	30891	4366	3956	5.51%	5.54%	0.03%
6	Uniform	105	30982	9666	71175	11540	8	24968	29294	8214	62531	12019	104	28992	29266	8110	61855	12057	1515	5.45%	5.54%	0.09%
	Gamma	105	29768	12332	97144	7630	8	21736	28704	10935	91297	7771	61	28292	28712	11024	91828	7333	1462	3.57%	3.55%	−0.03%
	Log-normal	105	28567	12790	96025	6844	8	18306	27898	11533	92674	8602	54	28032	27895	11469	92532	8495	1514	2.34%	2.35%	0.01%
7	Uniform	109	47532	24011	148256	7957	11	24553	30702	14628	100022	8217	188	29037	29040	13364	91849	7646	2643	35.41%	38.9%	5.41%
	Gamma	109	46884	27273	181327	7128	11	16240	31231	19506	143807	5779	104	28958	29688	17625	133735	6616	1482	33.39%	36.68%	4.94%
	Log-normal	109	45255	26677	154906	6307	11	16116	30553	18905	119795	6248	93	29178	29200	17070	111319	6404	2466	32.49%	35.48%	4.43%
8	Uniform	110	117571	59582	360044	20530	10	50454	94951	46082	295601	20005	75	88434	90101	41797	275842	21556	2715	19.24%	23.36%	5.11%
	Gamma	110	113859	71698	470329	12076	10	35842	94877	61306	422260	15549	51	90237	91162	55792	397385	17637	2018	16.67%	19.93%	3.92%
	Log-normal	110	107695	73464	494905	14110	10	35842	90785	63266	447196	14242	51	88003	87779	59016	429276	14092	1846	15.7%	18.49%	3.31%
Average		106	51011	27848	181623	9033	9	33175	35072	16734	128628	10200	42	33037	34333	15893	124848	10187	1847	31.25%	32.69%	2.11%

Table 4.2 – Performance of the deterministic, stochastic and robust models focused on minimum average cost.

		Deterministic						Robust						Stochastic						Worst-case Difference		
Class #1	Distribution #2	Obj. Function #3	Average #4	Std Dev. #5	Worst-case #6	Best-case #7	Runtime(s) #8	Obj. Function #9	Average #10	Std Dev. #11	Worst-case #12	Best-case #13	Runtime(s) #14	Obj. Function #15	Average #16	Std Dev. #17	Worst-case #18	Best-case #19	Runtime(s) #20	Det−Robust #21	Det−Stoch #22	Robust−Stoch #23
1	Uniform	117	14917	4535	33684	5938	12	26975	13574	3344	25225	5361	16	13489	13542	3397	25423	5455	4846	25.11%	24.53%	−0.78%
	Gamma	117	14548	5511	40207	4314	12	43322	14954	3917	29375	5133	12	13470	13674	4243	32506	4497	599	26.94%	19.15%	−10.66%
	Log-normal	117	14436	5419	36392	4349	12	45286	15465	3895	30207	6070	15	13470	13797	4129	32185	4827	599	17%	11.56%	−6.55%
2	Uniform	131	33257	10040	74788	12946	12	61968	30477	7465	57394	13129	16	29847	30293	7650	58444	11848	627	23.26%	21.85%	−1.83%
	Gamma	131	31991	12919	98631	7763	12	132641	39476	8701	72938	18135	16	29942	30821	9923	84450	11303	747	26.05%	14.38%	−15.78%
	Log-normal	131	30807	13435	102458	7538	12	132641	40283	8849	85611	16806	16	29847	30702	10349	91333	10284	627	16.44%	10.86%	−6.68%
3	Uniform	84	45216	23287	141329	7829	7	41868	19955	4107	33212	7610	9	17034	17265	4893	42089	7226	397	76.5%	70.22%	−26.73%
	Gamma	84	44461	26413	167418	6619	7	62834	28515	4299	42066	14493	10	14746	20566	7872	70681	8583	9	74.87%	57.78%	−68.02%
	Log-normal	84	43012	26042	146766	6188	7	65076	29857	4278	45574	17221	8	14746	20821	8016	76563	8900	9	68.95%	47.83%	−68%
4	Uniform	108	118430	60681	374027	19806	9	112720	51787	10583	86739	18712	140	53296	45370	12766	110896	17318	91	76.81%	70.35%	−27.85%
	Gamma	108	114599	72841	481681	12005	9	205688	73486	19991	184853	34793	85	43643	60905	26870	254147	25403	10	61.62%	47.24%	−37.49%
	Log-normal	108	108565	74862	479698	13396	9	229810	82099	22298	239944	40366	76	53569	56100	30973	297388	21319	42	49.98%	38.01%	−23.94%
5	Uniform	78	14289	4397	32626	5612	10	23727	13188	3330	24574	5800	67	12984	13075	3382	25275	5578	272	24.68%	22.53%	−2.85%
	Gamma	78	13907	5308	40672	4175	10	46113	16007	4118	31804	6035	73	12836	12974	4249	34224	4197	121	21.8%	15.85%	−7.61%
	Log-normal	78	13698	5197	34454	4171	10	23727	13158	4011	30359	4588	67	12984	12987	4102	30614	4565	272	11.88%	11.15%	−0.84%
6	Uniform	105	30982	9666	71175	11540	8	61023	31488	7732	57854	13222	97	28674	29324	8113	61573	11786	305	18.72%	13.49%	−6.43%
	Gamma	105	29768	12332	97144	7630	8	127878	42330	8979	84357	20442	58	28772	28779	10648	89856	8800	1083	13.16%	7.5%	−6.52%
	Log-normal	105	28567	12790	96025	6844	8	34198	28520	10890	91920	9880	115	26046	29961	11315	90148	10435	26	4.27%	6.12%	1.93%
7	Uniform	109	47532	24011	148256	7957	11	71778	46130	11369	84971	18958	83	28272	29336	13417	91659	7709	164	42.69%	38.17%	−7.87%
	Gamma	109	46884	27273	181327	7128	11	110336	67692	11669	114327	40734	75	28272	30121	17081	129631	7559	164	36.95%	28.51%	−13.39%
	Log-normal	109	45255	26677	154906	6307	11	41882	34063	16010	114127	9605	90	29037	29346	16647	109447	6640	2643	26.32%	29.35%	4.1%
8	Uniform	110	117571	59582	360044	20530	10	213187	131852	34307	241753	53448	59	88940	90275	41326	271999	21358	590	32.85%	24.45%	−12.51%
	Gamma	110	113859	71698	470329	12076	10	230860	140607	44043	379875	59275	46	88940	91741	54543	391945	18412	590	19.23%	16.67%	−3.18%
	Log-normal	110	107695	73464	494905	14110	10	268204	147058	44674	425204	69074	41	89245	88376	56907	421613	16942	1166	14.08%	14.81%	0.84%
Average		106	51011	27848	181623	9033	9	100572	48001	12618	108928	21203	54	33422	35006	15533	121838	10873	667	40.03%	32.92%	−11.85%

Table 4.3 – Performance of the deterministic, stochastic and robust models focused on minimum worst-case.

4.4.2.2 Analysis of solutions characteristics

The histograms in Figure 4.7 exhibit the probability of incurring backorders, which is evaluated as the total number of times when we have a backlogging in the last period over the 5 products and the 10,000 simulation runs, totaling 50,000 possibilities. As expected, the deterministic model delivers the overall worst performance. Notice that backorder occurs at least 44% of the times with probability up to 0.33 and more than 48% of the times with probability of 0.5. These results jointly with the average low cost performance of the deterministic model corroborate its inability to hedge against uncertainty demand. Although both RO and SP models provide good backlog performances, RO with the best combination found to minimize worst-case values (RO-WC) clearly dominates SP in terms of average (20.92% against 27.97%), maximum (41.24% against 42.62%), and minimum backlogging values (0.14% against 8.56%). It is remarkable that RO-WC yields solutions that may generate a backordered demand for any product less than 1% of the times with almost 0.17 of probability.

A deeper analysis into the backlogging and its corresponding cost brings out differences between RO and SP. Both approaches present a similar solution structure in many instances when the best combination of parameters is focused on minimizing average costs. In fact, the first graph depicted in Figure 4.8 shows a clear concentration of solutions around zero and that most values are within 4%, indicating that, in these cases, the relative difference in terms of backlog quantities and costs between both models is negligible. Moreover, RO dominates SP in both criteria, backlog quantities and backlog costs, for all the solutions in the shaded area. Notice that RO is more effective to mitigate the backlogging because most solutions are on the left of zero, but SP is more effective to minimize backlogging costs, since most solutions are above zero. On average, RO is able to fulfill 0.2% more products than SP, but the latter saves 3.4% in backlogging costs. When the best combination of parameters focuses on minimizing worst-case costs, though, the difference between RO and SP solutions is much more pronounced. In this case, RO clearly dominates SP for a wider range of solutions. On average, RO is able to fulfill 114% more products with a corresponding cost 149% cheaper in comparison to SP.

Figures 4.9 and 4.10 illustrate the average backlogging in the last period for both RO and SP models in classes 1 and 8, respectively. When production capacity is not too restricted (class 1), backlogging is substantially reduced as robustness is enforced, i.e., via more conservative budgets and higher variability levels. However, when production capacity is too restricted (class 8), conservative solutions try to protect almost integrally all worst-case demand deviations, which is not possible due to the limited capacity, which deteriorates backlogging levels. In fact, the so-called over conservativeness of the RO together with some characteristics of the problem/instance might lead to poor solution quality in this case. If $\Gamma_{jt} = t$, for instance, the demand constraint is "fully protected" because of the cumulative effect of aggregating the fully deviation $\hat{d}_{jt}$ to the nominal value d_{jt}. In the last period, the demand deviations from all periods, i.e., $\hat{d}_{j|T|}$, $\hat{d}_{j|T|-1}$, $\hat{d}_{j|T|-2}, \cdots, \hat{d}_{j1}$, are incorporated in the nominal value, resulting pessimistic solutions in some cases. This behavior was already pointed out in other papers focused on different applications, e.g., Alem and Morabito (2012); Gorissen and den Hertog (2013); Thorsen and Yao (2015).

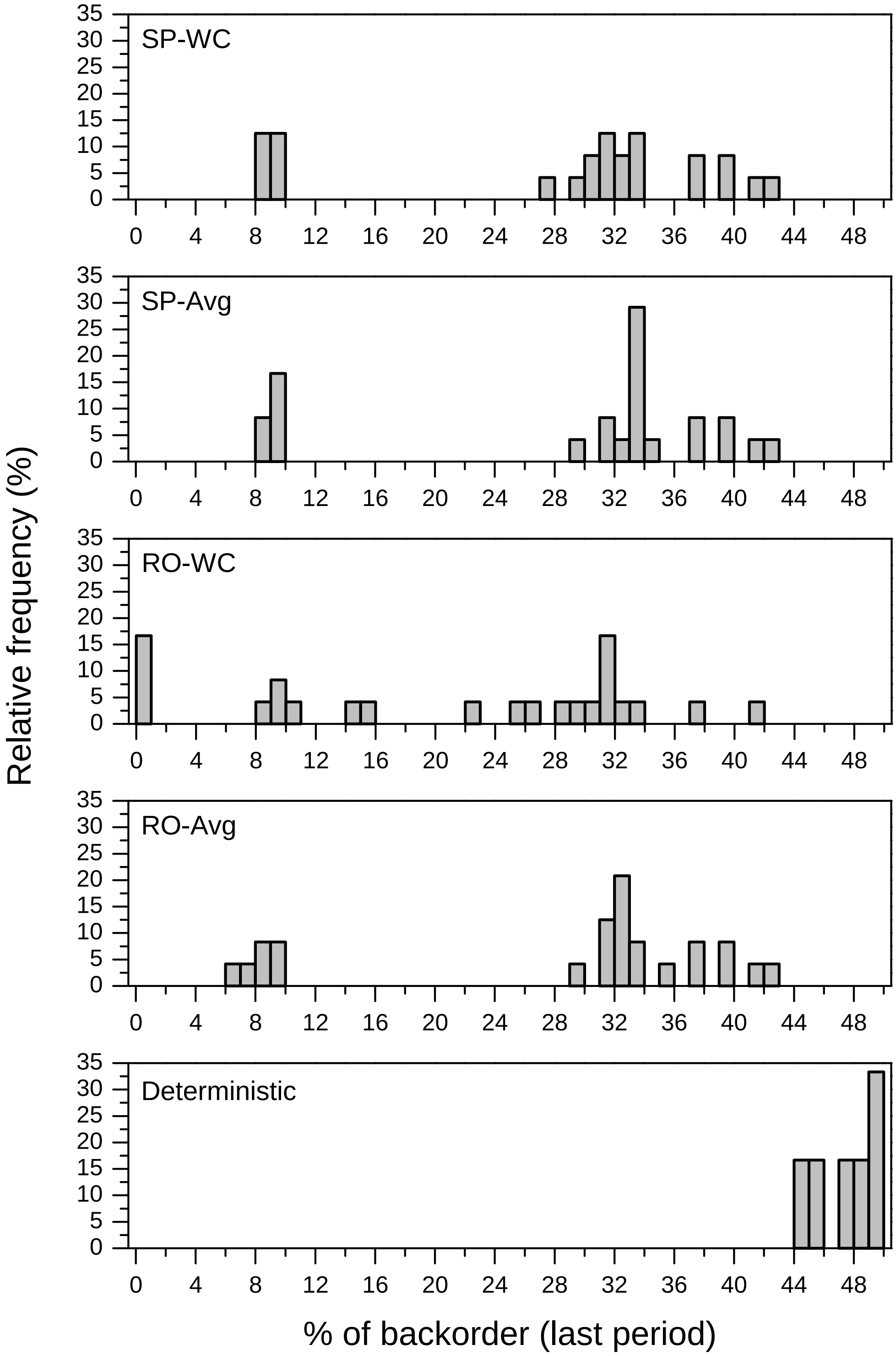


Figure 4.7 – Histograms of the backorder in the last period for all approaches: deterministic, RO focusing on average values (RO-Avg), RO focusing on worst-case values (RO-WC), SP focusing on average values (SP-Avg), and SP focusing on worst-case values (SP-WC).

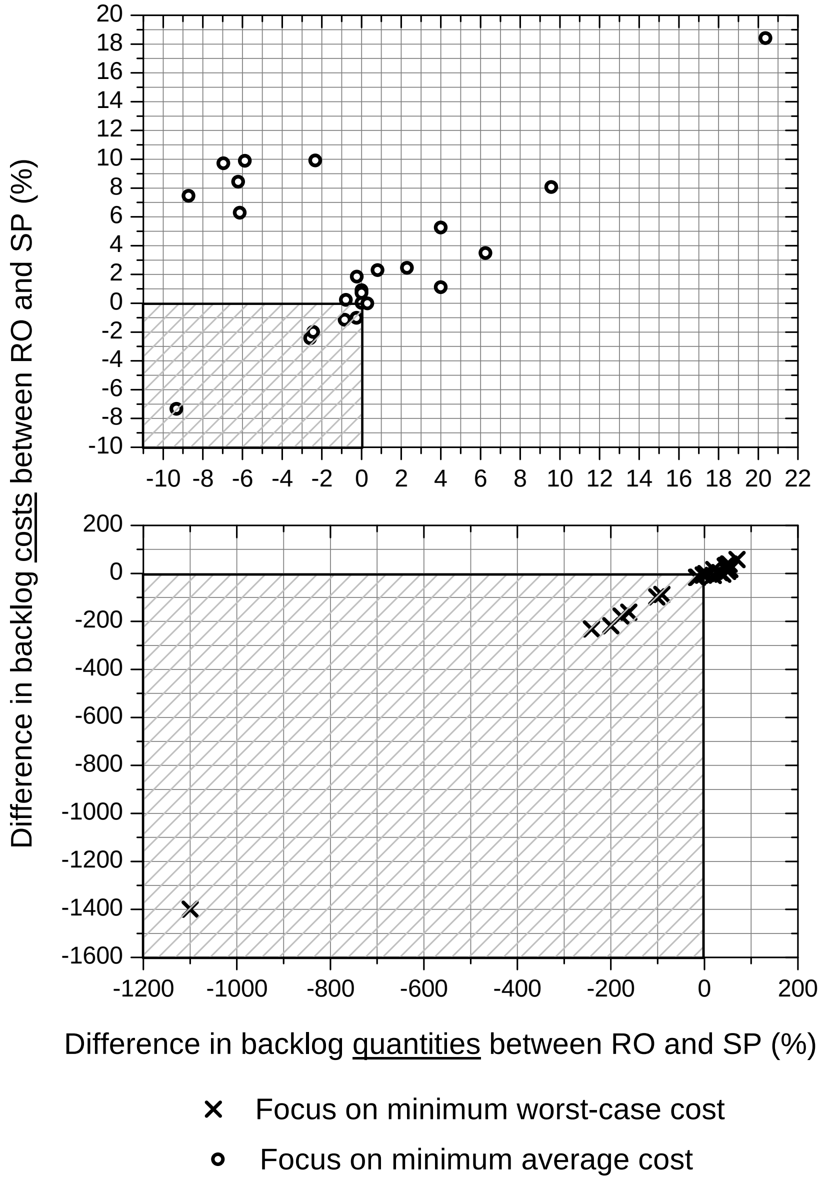
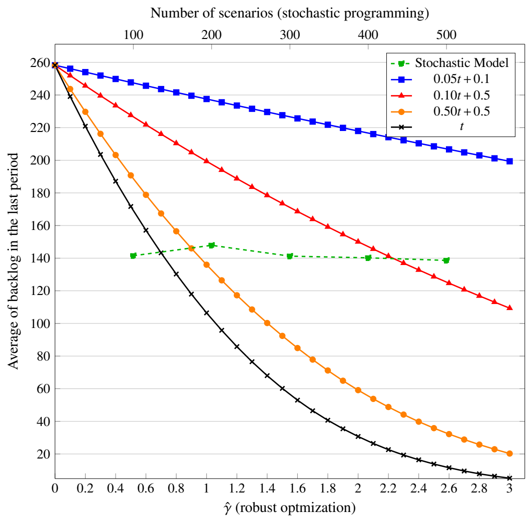
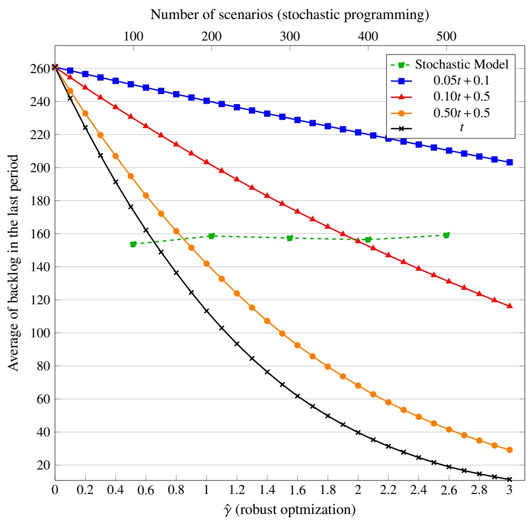


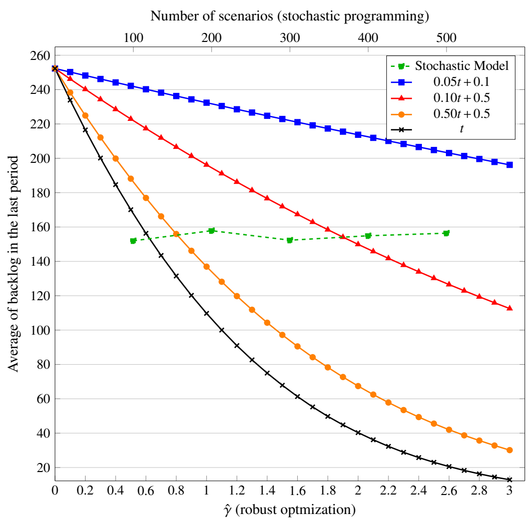
Figure 4.8 – Tradeoffs in terms of backlog quantities and costs between RO and SP. Negative differences indicate that the RO model leads to a better solution in terms of backlogging quantities and/or backlogging costs. The above graph refers to the values that lead to the best combination of parameters to minimize average costs (Table 4.8) and the below graph refers to the values that lead to the best combination of parameters to minimize worst-case costs (Table 4.9).



(a) Uniform demand distribution.

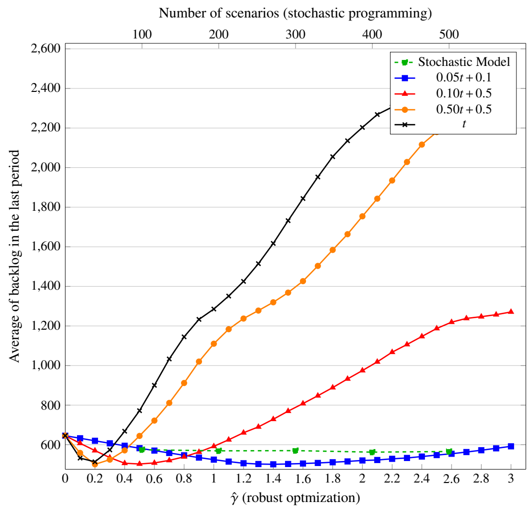


(b) Gamma demand distribution.

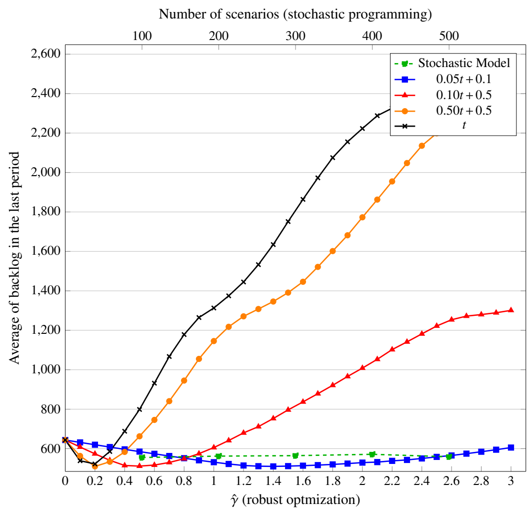


(c) Log-normal demand distribution.

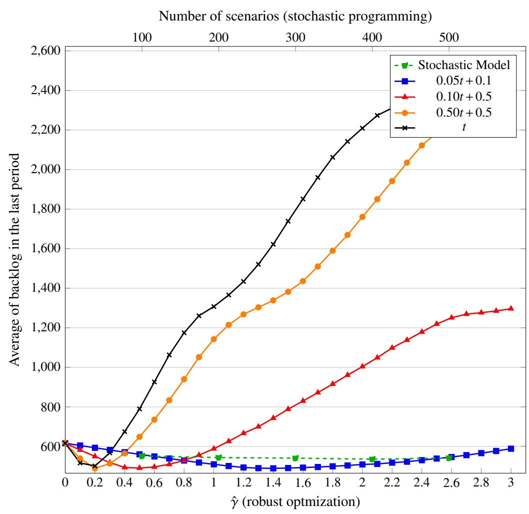
Figure 4.9 – Robust optimization and stochastic programming final backlog for the instance class 1.



(a) Uniform demand distribution.



(b) Gamma demand distribution.



(c) Log-normal demand distribution.

Figure 4.10 – Robust optimization and stochastic programming final backlog for the instance class 8.

Both stochastic and robust models present similar average inventory levels and costs, which are both higher than those from the deterministic model, as illustrated in Figure 4.11. In particular, it is worth commenting that class 4 presents a substantially larger amount of inventory in all approaches, probably as consequence of its higher coefficient of variation, greater capacity, and relatively cheaper inventory cost. This behavior is more pronounced in RO focusing on worst-case values. The results also show that RO-WC maintains a relatively higher amount of inventory with fewer setups in comparison to the remaining approaches, probably in an attempt to meet worst-case demands and avoid backordering. However, this strategy sometimes fails due to the over protection, as previously discussed.

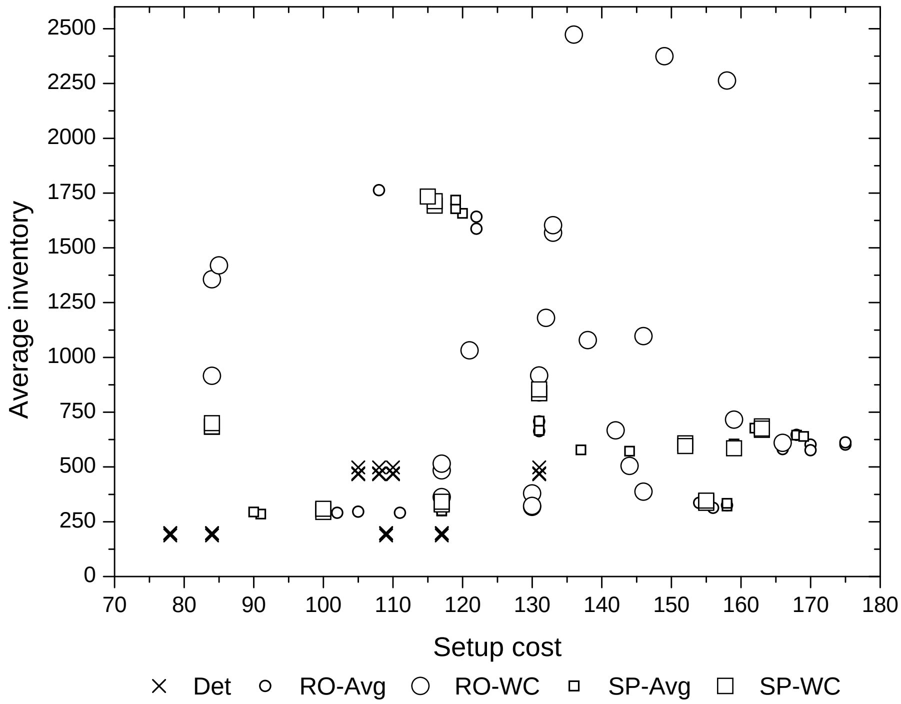


Figure 4.11 – Tradeoffs between average inventory and setup costs for all approaches: deterministic, RO focusing on average values (RO-Avg), RO focusing on worst-case values (RO-WC), SP focusing on average values (SP-Avg), and SP focusing on worst-case values (SP-WC).

4.4.2.3 Guidelines for decision makers

Based on the insights of our extensive numerical results, we propose a general flowchart to assist decision makers to select the most appropriate modeling approach to deal with optimization under uncertainty. This is accomplished via the development of the decision tree depicted in Figure 4.12, which is based on three main questions:

1. Is it possible to generate plausible scenarios?
2. Is the time required to obtain a solution critical?

3. Is it possible to test more than one approach and decide a *posteriori* which solution will be implemented?

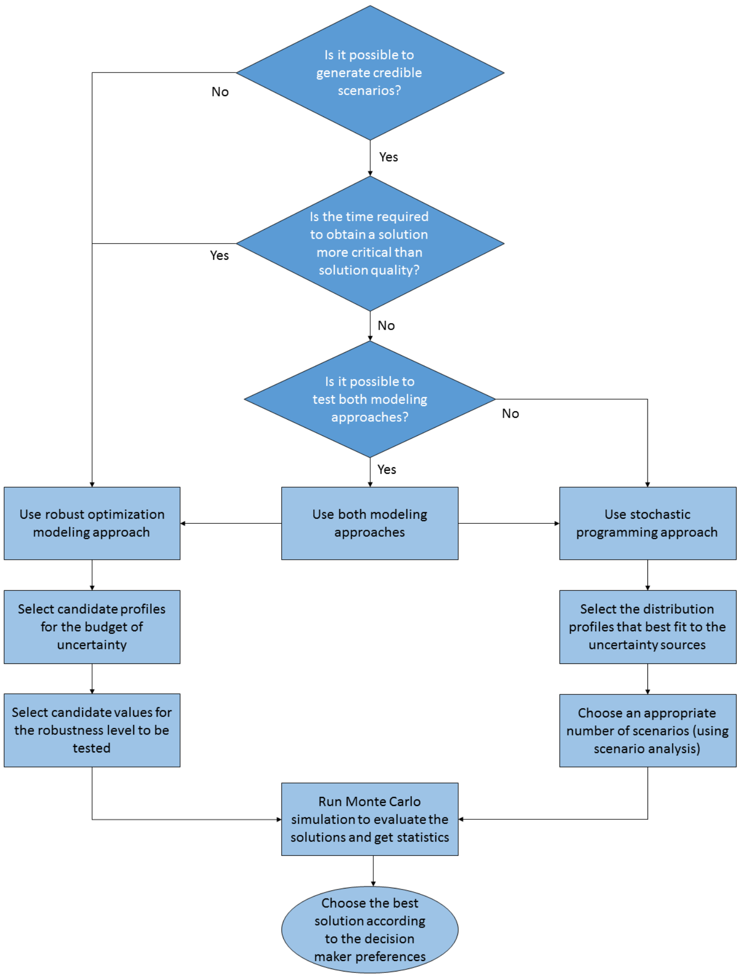


Figure 4.12 – General flowchart for decision makers to select the uncertainty modeling approach.

Although there is a variety of well-established scenario generation methods in the literature, there are none unrestrictedly recommendable ones for all possible mathematical models, even if these models are subject to the same random phenomena (Kaut and Wallace, 2003). Most methods are based on the existence of sufficient data to match statistical properties, which is not a trivial assumption in real-life applications; others, though, simply assume a *priori* underlying distribution without any further tests to analyze the quality of the generated scenarios. Therefore, providing a reasonable set of scenarios is usually a challenging task. Obviously, if the answer for the first question is “no”, RO must be used to hedge against the uncertainty. In this case, it will be necessary to select a budget of uncertainty and a variability level. We have discussed along the paper that suitable budgets are chosen according to risk preferences, simulation, and/or probabilities of constraints violation equipped with a confidence level, which should be intuitive for most decision makers. The variability level also does not require (precise) data; it might reflect a hypothetical deviation from the nominal data and it can also be estimated via simulation in the absence of further information.

The second question refers to the well-known tradeoff between available time and solution quality. Notice that it is necessary to define what is an acceptable runtime to further evaluate if this time is critical or not. It could be acceptable to run strategic models within many hours, but tactical and operational models often must be solved within minutes or even seconds. In general, as stochastic programming models require a relatively large set of scenarios to provide accurate solutions, usually at the expense of an expensive computational burden, RO must be preferred if the time required to obtain a solution is critical.

It is also important to notice that tactical production planning does not always require fast and agile solutions, and depending on the number of products, length of the time period, instance size and structure, the time required to solve the models can be suitable for both modeling approaches. In addition, even if the robust optimization model is much more tractable, it may require tuning and testing the robust parameters, which can increase the time required to reach good solutions. However, there are strategies to work around this issue, such as using previous historical data to define the variability level, testing only the combinations of robust parameters that had good performance in similar cases in the past, and defining the budget of uncertainty using the theoretical probability bounds on the constraints violation according to inequalities (4.23) and (4.24), which eliminates the necessity of testing several profiles of budgets of uncertainty. For example, Figure 4.13 shows that the probability of constraint violation decreases as time period increases because budgets of uncertainty are time-dependent. As robustness is enforced, via more conservative budgets, this probability decreases convexly and tends to reach zero nearly after the end of the time horizon. Notice also that even though less conservative budgets lead to high probabilities of constraint violation in a short-term horizon, the empirical performance of such budgets is very good.

The third question is related to the possibility of testing both modeling approaches. In effect, if it is possible to supply plausible scenarios and the time required to obtain a solution is not critical, SP could be a more “natural” fashion to represent and solve the optimization problem under uncertainty. However, even in this case, RO has a great potential to generate alternative good solutions from a worst-case perspective mainly, which in turn usually results in more stable solutions. For this reason, we claim in this paper that using both modeling paradigms provide a more precise manner to make decisions when uncertainty matters. If both approaches are adopted, decision-makers may run a Monte Carlo simulation to get useful statistics and implement solutions according to their preferences.

The flowchart depicted in Figure 4.14 shows how to empirically select the most appropriate modeling approach to deal with the GLSP model under demand uncertainty, without needing to perform a Monte Carlo simulation experiment. We assume that the decision-maker knows the mean and the coefficient of variation of the demand, the relation between costs and capacities as well. The criteria used to select the modeling approach are faster runtimes and whether the average cost of the modeling performance is the best or is within

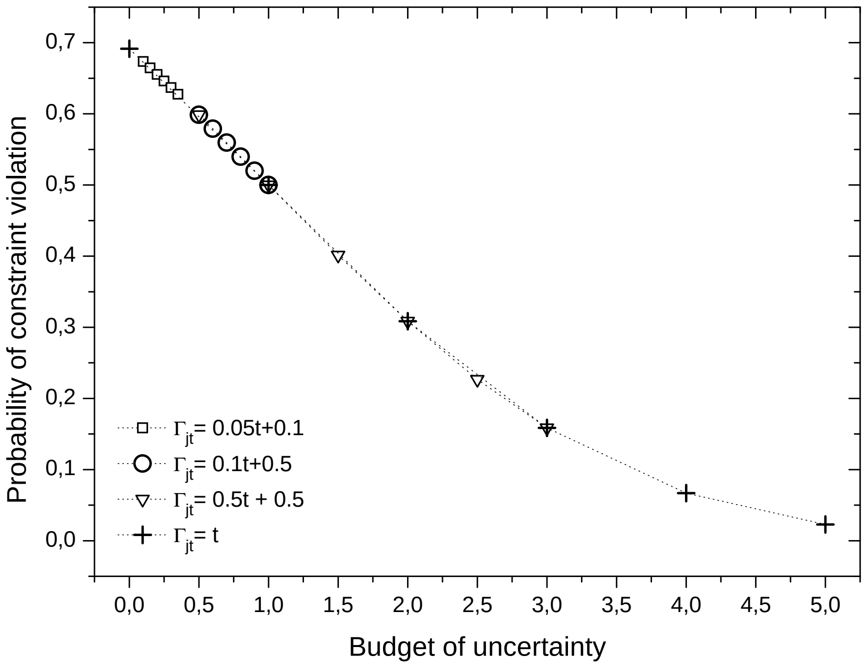


Figure 4.13 – Theoretical probabilities of constraint violation for each proposed budget of uncertainty.

the 2% of the best modeling approach performance. If there is a case in which the parameters characteristics are different from the instance classes we proposed, the best option is to select the strategy for an instance class that has similar parameters characteristics.

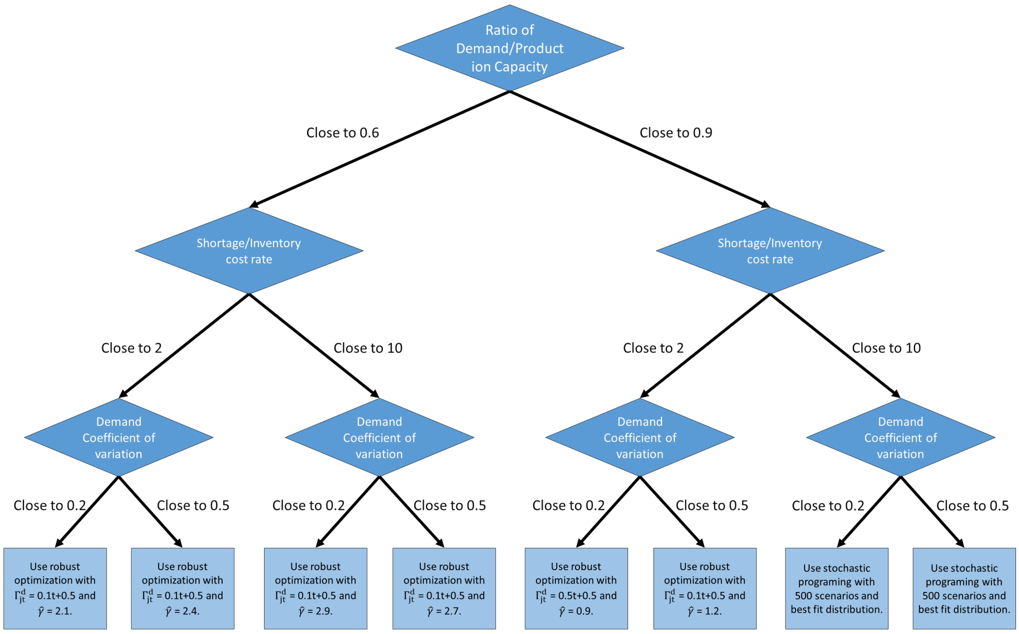


Figure 4.14 – Specific flowchart for decision makers to select the uncertainty modeling approach without running the simulation experiment.

As mentioned before, the demand distribution does not have a significant impact on the performance of the best combination of Γ_{jt} and $\hat{\gamma}$ for the robust model. Nevertheless, in cases where costs are very sensitive, it is possible to select the best combination of robust parameters for each class of instance and demand distribution using the Tables 4.5 and/or 4.6. If necessary, this flowchart can be extended to a more fine discretization and/or additional instance characteristics for a more precise selection of modeling approach and uncertainty parameters. Furthermore, we can also incorporate risk preferences, e.g., if the decison-maker is more average- or worst-case oriented so as to distinguish the type of solution that should be preferred in each case.

4.5. Conclusion

This paper presented a "distribution-free" robust optimization and a scenario-based two-stage stochastic programming model to deal with the GLSP model under demand uncertainty. Although both methodologies are popular in production planning problems, this paper arises as the first effort in using both of them in a specific lot-sizing and scheduling problem. Because of the lack of a systematic methodology to assess the advantages and disadvantages of each modeling approach, we proposed an extensive simulation experiment based on Monte Carlo to evaluate different characteristics of the solutions, such as average costs, worst-case costs, and standard deviation. The extensive numerical study clearly highlighted the importance of the simulation experiment for tuning the parameters of both approaches and to select an appropriate modeling approach aligned to the decision makers preferences and goals. The main results showed that both approaches outperform the deterministic model at the expense of a minor increase in price. Also, we confirmed the main known tradeoffs among robust optimization and stochastic programming in terms of solution quality and runtime, and provided some new insights allowing decision-makers to choose the best solution according to their preferences and for different instance structures of the GLSP. The proposed flowcharts help decision makers in this task, without necessarily needing to run a simulation experiment. Contrary to common sense, our overall findings indicate that the RO model has not only the best worst-case performance, but also a good average performance. On the other hand, the SP model rarely presents a good worst-case performance. Future research relies on the study of the GLSP under endogenous uncertainty sources, such as processing and setup times. In addition, the possibility of readjusting the production in each time period via multi-stage stochastic programming and/or adjustable robust optimization is also a promising topic for further research.

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Appendix 4.A Supplementary material

	Assumed Distribution	Uniform				Gamma				Log-normal				Assumed Distributions Difference
ID	Real Distribution	Average	Std. Deviation	Worst-case	Best-case	Average	Std. Deviation	Worst-case	Best-case	Average	Std. Deviation	Worst-case	Best-case	
1	Uniform	13541	3381	25461	5488	13606	3478	26405	5821	13618	3484	26627	5853	0.57%
	Gamma	13561	4285	32965	4644	13520	4418	34165	4190	13526	4462	34214	4123	0.3%
	Log-normal	13701	4172	32417	4823	13613	4307	32981	4718	13607	4369	33397	4748	0.69%
2	Uniform	30225	7604	58883	12066	30543	7968	62475	13311	30876	8264	64529	13546	2.11%
	Gamma	30762	9913	85580	11328	30425	10622	88525	9645	30535	10999	90165	8692	1.1%
	Log-normal	30690	10324	91831	11046	29976	11170	95475	9935	29935	11543	96082	9924	2.46%
3	Uniform	17265	4893	42089	7226	17321	5125	43829	7190	17290	5001	43051	7061	0.32%
	Gamma	19238	7981	74931	7128	19286	8399	76767	6970	19262	8170	75940	7243	0.25%
	Log-normal	19542	8171	77489	8072	19576	8601	79585	7728	19566	8391	79122	8040	0.17%
4	Uniform	45068	12941	116149	17497	45170	12884	115840	17693	45204	12677	111515	17786	0.3%
	Gamma	54088	27142	270554	21406	54135	26911	268051	21634	54147	26609	266882	21163	0.11%
	Log-normal	55796	31232	301876	21661	55866	31073	302785	21082	55921	30775	300463	21952	0.22%
5	Uniform	13075	3382	25275	5578	13092	3440	26082	5710	13103	3454	26292	5594	0.21%
	Gamma	12935	4223	34253	3915	12924	4318	34945	3688	12936	4337	35259	3616	0.09%
	Log-normal	12987	4102	30614	4565	12943	4197	31067	4507	12939	4205	30891	4366	0.37%
6	Uniform	29266	8110	61855	12057	29366	8361	63339	11739	29392	8424	63752	11686	0.43%
	Gamma	28762	10699	90190	8592	28712	11024	91828	7333	28716	11037	91868	7285	0.18%
	Log-normal	28102	11115	91618	9239	27901	11438	92416	8740	27895	11469	92532	8495	0.74%
7	Uniform	29040	13364	91849	7646	29146	13768	94786	7674	29155	13755	94464	7489	0.4%
	Gamma	29771	17087	130561	7137	29688	17625	133735	6616	29689	17558	133006	6614	0.28%
	Log-normal	29346	16647	109447	6640	29200	17070	111319	6404	29237	17112	111900	6318	0.5%
8	Uniform	90101	41797	275842	21556	90306	42479	279793	20784	90619	42617	281122	21110	0.57%
	Gamma	91409	55026	395941	18588	91162	55792	397385	17637	91339	56930	403201	16715	0.27%
	Log-normal	88287	57175	422622	16996	87857	58008	427282	15889	87779	59016	429276	14092	0.58%

Table 4.4 – Stochastic model performance.

ID	Distribution	Average	Std Dev.	Worst-case	Best-case	Γ_{jt}	$\hat{d}_{j\tau}$
1	Uniform	13534	3382	25488	5455	0.10t + 0.5	2.5
	Gamma	13498	4385	33775	4257	0.10t + 0.5	2.1
	Log-normal	13588	4352	33117	4861	0.10t + 0.5	1.9
2	Uniform	30218	7622	59064	12268	0.10t + 0.5	2.4
	Gamma	30397	10668	89382	9535	0.10t + 0.5	1.7
	Log-normal	29882	11516	95921	10409	t	0.5
3	Uniform	17403	5012	43755	7683	0.10t + 0.5	2.9
	Gamma	19317	8043	75304	7252	0.10t + 0.5	2.9
	Log-normal	19599	8093	77208	7810	0.10t + 0.5	2.9
4	Uniform	45715	14364	123259	17978	0.10t + 0.5	2.7
	Gamma	54509	25396	260755	23617	0.50t + 0.5	1.4
	Log-normal	56052	32406	302733	21890	0.10t + 0.5	2.7
5	Uniform	13093	3445	25896	5372	0.10t + 0.5	1.9
	Gamma	12930	4298	34953	3660	0.50t + 0.5	0.9
	Log-normal	12943	4259	31281	4342	0.50t + 0.5	0.8
6	Uniform	29294	8214	62531	12019	0.10t + 0.5	1.2
	Gamma	28704	10935	91297	7771	0.50t + 0.5	0.5
	Log-normal	27898	11533	92674	8602	0.05t + 0.1	3
7	Uniform	30702	14628	100022	8217	0.10t + 0.5	1.3
	Gamma	31231	19506	143807	5779	0.50t + 0.5	0.5
	Log-normal	30553	18905	119795	6248	t	0.4
8	Uniform	94951	46082	295601	20005	0.10t + 0.5	0.8
	Gamma	94877	61306	422260	15549	0.50t + 0.5	0.3
	Log-normal	90785	63266	447196	14242	0.50t + 0.5	0.3

Table 4.5 – Robust model performance focused on minimizing average cost.

ID	Distribution	Average	Std Dev.	Worst-case	Best-case	Γ_{jt}	$\hat{d}_{j\tau}$
1	Uniform	13574	3344	25225	5361	0.10t + 0.5	2.9
	Gamma	14954	3917	29375	5133	0.50t + 0.5	2.2
	Log-normal	15465	3895	30207	6070	0.50t + 0.5	2.3
2	Uniform	30477	7465	57394	13129	0.50t + 0.5	1.4
	Gamma	39476	8701	72938	18135	0.50t + 0.5	3
	Log-normal	40283	8849	85611	16806	0.50t + 0.5	3
3	Uniform	19955	4107	33212	7610	t	1.5
	Gamma	28515	4299	42066	14493	0.50t + 0.5	2.8
	Log-normal	29857	4278	45574	17221	0.50t + 0.5	2.9
4	Uniform	51787	10583	86739	18712	t	1.5
	Gamma	73486	19991	184853	34793	t	1.9
	Log-normal	82099	22298	239944	40366	0.50t + 0.5	2.5
5	Uniform	13188	3330	24574	5800	0.50t + 0.5	1.2
	Gamma	16007	4118	31804	6035	t	1.7
	Log-normal	13158	4011	30359	4588	0.50t + 0.5	1.2
6	Uniform	31488	7732	57854	13222	0.50t + 0.5	1.3
	Gamma	42330	8979	84357	20442	t	2.1
	Log-normal	28520	10890	91920	9880	0.10t + 0.5	1.6
7	Uniform	46130	11369	84971	18958	t	1
	Gamma	67692	11669	114327	40734	t	1.4
	Log-normal	34063	16010	114127	9605	0.10t + 0.5	1.8
8	Uniform	131852	34307	241753	53448	0.10t + 0.5	2.5
	Gamma	140607	44043	379875	59275	t	1
	Log-normal	147058	44674	425204	69074	0.10t + 0.5	3

Table 4.6 – Robust model performance focused on minimizing worst-case scenario.

		Robust					Stochastic					Cost Difference
Class	Distribution	Average	Std Dev.	Worst-case	Best-case	Runtime(s)	Average	Std Dev.	Worst-case	Best-case	Runtime(s)	Robust-Stoch
1	Uniform	13534	3382	25488	5455	12	14625	3768	29178	5979	55	-8.05%
	Gamma	13498	4385	33775	4257	14	14414	4719	36317	5150	57	-6.78%
	Log-normal	13588	4352	33117	4861	11	14754	4991	35621	5169	47	-8.58%
2	Uniform	30218	7622	59064	12268	14	32598	8643	65756	12393	17	-7.88%
	Gamma	30397	10668	89382	9535	19	32980	11907	94391	10920	24	-8.5%
	Log-normal	29882	11516	95921	10409	11	31815	11643	99974	11127	34	-6.47%
3	Uniform	17403	5012	43755	7683	9	18870	5306	45620	7658	9	-8.43%
	Gamma	19317	8043	75304	7252	9	21225	8621	77780	8619	7	-9.88%
	Log-normal	19599	8093	77208	7810	9	22593	9430	77493	9595	8	-15.28%
4	Uniform	45715	14364	123259	17978	10	51092	16384	134645	20281	7	-11.76%
	Gamma	54509	25396	260755	23617	37	60905	26870	254147	25403	10	-11.73%
	Log-normal	56052	32406	302733	21890	10	67891	35366	305136	28424	9	-21.12%
5	Uniform	13093	3445	25896	5372	15	13867	3708	27006	5701	17	-5.91%
	Gamma	12930	4298	34953	3660	35	13693	4583	36363	4785	30	-5.9%
	Log-normal	12943	4259	31281	4342	15	13997	4410	33343	4974	22	-8.14%
6	Uniform	29294	8214	62531	12019	104	31017	8764	63458	12415	26	-5.88%
	Gamma	28704	10935	91297	7771	61	30067	11032	92464	9861	23	-4.75%
	Log-normal	27898	11533	92674	8602	54	29597	11679	94358	9612	28	-6.09%
7	Uniform	30702	14628	100022	8217	188	31194	14381	95816	8037	24	-1.6%
	Gamma	31231	19506	143807	5779	104	32514	18424	132964	7627	25	-4.11%
	Log-normal	30553	18905	119795	6248	93	32052	18387	116517	7289	20	-4.91%
8	Uniform	94951	46082	295601	20005	75	96008	42712	277813	22646	27	-1.11%
	Gamma	94877	61306	422260	15549	51	98219	58107	410151	18153	20	-3.52%
	Log-normal	90785	63266	447196	14242	51	92873	59944	425329	17221	19	-2.3%
Average		35069.7	16734.1	128628	10200.8	42.1	37452.5	16824.1	127568	11626.7	23.6	-7.45%

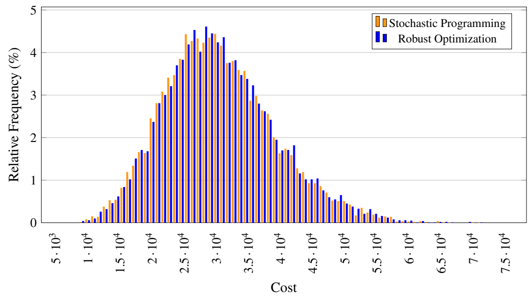
Table 4.7 – Performance of the stochastic (with 10 scenarios) and robust models.

		Deterministic							Robust							Stochastic						
Class	Distribution	Avg. Backlog	Avg. Inventory	% of Backlog (last period)	Qt. of Backlog (last period)	Backlog Cost	Inv. Cost	Setup Cost	Avg. Backlog	Avg. Inventory	% of Backlog (last period)	Qt. of Backlog (last period)	Backlog Cost	Inv. Cost	Setup Cost	Avg. Backlog	Avg. Inventory	% of Backlog (last period)	Qt. of Backlog (last period)	Backlog Cost	Inv. Cost	Setup Cost
	Uniform	198	199	49.14	258	9861	4940	117	102	336	31.28	129	5080	8337	117	102	337	32.94	139	5034	8390	117
1	Gamma	195	188	48.7	261	9736	4696	117	116	305	32.76	151	5794	7587	117	119	299	33.44	156	5935	7468	117
	Log-normal	188	196	47.42	252	9430	4888	117	118	303	32.94	154	5933	7539	117	119	300	33.34	156	6001	7489	117
	Uniform	495	498	49.28	646	21992	11134	131	262	826	32.16	332	11639	18448	131	256	837	33.38	352	11353	18742	131
2	Gamma	481	467	45.76	644	21423	10437	131	325	708	33.74	426	14462	15804	131	325	709	33.88	425	14459	15835	131
	Log-normal	453	471	44.34	616	20184	10492	131	336	663	32.46	427	14965	14786	131	335	667	33.1	437	14966	14839	131
	Uniform	198	199	49.14	258	41008	4123	84	16	682	7.84	19	3129	14189	84	15	683	8.56	22	3020	14161	84
3	Gamma	195	188	48.7	261	40501	3876	84	25	683	8.36	28	5048	14185	84	24	684	9.16	30	4992	14162	84
	Log-normal	188	196	47.42	252	38877	4050	84	24	698	8.08	29	4998	14517	84	24	700	8.8	31	4962	14496	84
	Uniform	495	498	49.28	646	107616	10706	108	54	1587	9.5	61	11089	34505	122	43	1657	9.18	59	9047	35902	120
4	Gamma	481	467	45.76	644	104410	10081	108	75	1763	6.86	64	16318	38083	108	82	1678	9.92	97	17514	36454	119
	Log-normal	453	471	44.34	616	98270	10188	108	94	1642	9.8	111	20108	35823	122	85	1718	9.5	109	18485	37192	119
	Uniform	198	199	49.14	258	9458	4752	78	125	296	35.1	155	5890	7098	105	120	305	33.8	145	5580	7395	100
5	Gamma	195	188	48.7	261	9356	4473	78	123	291	32.86	152	5871	6948	111	126	285	34.08	160	5988	6846	91
	Log-normal	188	196	47.42	252	8948	4672	78	124	291	33.4	156	5911	6930	102	123	294	33.52	158	5775	7074	90
	Uniform	495	498	49.28	646	20537	10340	105	416	602	42.72	516	16002	13123	170	417	605	42.62	517	15706	13401	159
6	Gamma	481	467	45.76	644	20005	9659	105	407	576	39.2	524	16158	12377	170	408	572	39.56	527	16321	12247	144
	Log-normal	453	471	44.34	616	18711	9752	105	382	581	37.04	502	15438	12295	166	385	578	37.46	507	15402	12355	137
	Uniform	198	199	49.14	258	43087	4335	109	129	336	31.72	137	22720	7828	154	132	340	31.78	142	20467	8418	155
7	Gamma	195	188	48.7	261	42701	4074	109	119	313	31.42	140	24009	7066	156	126	321	31.34	148	21635	7896	158
	Log-normal	188	196	47.42	252	40890	4256	109	115	325	29.46	138	23045	7351	158	123	334	29.98	144	20804	8238	158
	Uniform	495	498	49.28	646	106683	10778	110	434	646	41.78	539	79627	15156	168	461	677	41.82	566	72905	17034	162
8	Gamma	481	467	45.76	644	103628	10120	110	413	602	39.4	533	80792	13909	175	449	645	39.58	559	74767	16227	168
	Log-normal	453	471	44.34	616	97404	10181	110	391	612	37.06	513	76451	14158	175	415	639	37.62	541	71653	15957	169

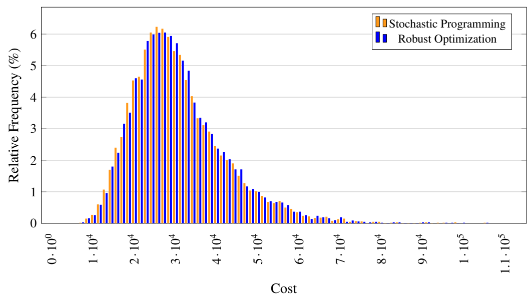
Table 4.8 – Comparison of solutions characteristics focused on minimum average values.

		Deterministic							Robust							Stochastic						
Class	Distribution	Avg. Backlog	Avg. Inventory	% of Backlog (last period)	Qt. of Backlog (last period)	Backlog Cost	Inv. Cost	Setup Cost	Avg. Backlog	Avg. Inventory	% of Backlog (last period)	Qt. of Backlog (last period)	Backlog Cost	Inv. Cost	Setup Cost	Avg. Backlog	Avg. Inventory	% of Backlog (last period)	Qt. of Backlog (last period)	Backlog Cost	Inv. Cost	Setup Cost
#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14	#15	#16	#17	#18	#19	#20	#21	#22	#23
	Uniform	198	199	49.14	258	9861	4940	117	90	362	28.76	113	4489	8968	117	103	333	33.1	140	5201	8224	117
1	Gamma	195	188	48.7	261	9736	4696	117	56	484	15.48	58	2806	12032	117	107	329	32.22	147	5260	8297	117
	Log-normal	188	196	47.42	252	9430	4888	117	51	515	14.04	53	2588	12759	117	103	341	30.9	142	5112	8568	117
	Uniform	495	498	49.28	646	21992	11134	131	221	917	26.84	251	9833	20514	131	258	837	32.7	342	11466	18697	131
2	Gamma	481	467	45.76	644	21423	10437	131	97	1569	9.28	90	4327	35016	133	270	836	30.58	370	12034	18656	131
	Log-normal	453	471	44.34	616	20184	10492	131	99	1603	8.92	101	4415	35735	133	259	854	27.48	353	11564	19008	131
	Uniform	198	199	49.14	258	41008	4123	84	5	916	0.68	1	950	18920	84	15	683	8.56	22	3020	14161	84
3	Gamma	195	188	48.7	261	40501	3876	84	2	1356	0.16	0	333	28097	84	24	684	9.16	30	4992	14162	84
	Log-normal	188	196	47.42	252	38877	4050	84	2	1420	0.14	1	331	29442	85	24	700	8.8	31	4962	14496	84
	Uniform	495	498	49.28	646	107616	10706	108	12	2263	0.8	2	2634	48995	158	41	1692	8.78	55	8733	36521	116
4	Gamma	481	467	45.76	644	104410	10081	108	130	2374	10.76	153	16556	56780	149	79	1713	9.7	94	17074	37051	116
	Log-normal	453	471	44.34	616	98270	10188	108	186	2473	9.66	138	22003	59960	136	85	1734	9.38	108	18396	37589	115
	Uniform	198	199	49.14	258	9458	4752	78	117	318	32.02	135	5205	7853	130	120	305	33.8	145	5580	7395	100
5	Gamma	195	188	48.7	261	9356	4473	78	183	379	31.08	258	5965	9913	130	123	294	33.38	156	5755	7119	100
	Log-normal	188	196	47.42	252	8948	4672	78	114	322	29.38	136	5069	7959	130	116	309	31.38	145	5405	7482	100
	Uniform	495	498	49.28	646	20537	10340	105	519	716	41.24	695	14390	16939	159	420	608	42.8	523	15596	13575	152
6	Gamma	481	467	45.76	644	20005	9659	105	821	1032	31.42	1240	17031	25178	121	413	585	39.52	527	15578	13042	159
	Log-normal	453	471	44.34	616	18711	9752	105	406	610	37.82	523	14472	13881	166	393	595	37.4	510	14622	13411	152
	Uniform	198	199	49.14	258	43087	4335	109	285	505	25.2	441	33600	12386	144	134	342	31.78	144	20690	8491	155
7	Gamma	195	188	48.7	261	42701	4074	109	451	667	22.16	725	51029	16521	142	136	336	31.2	149	21607	8360	155
	Log-normal	188	196	47.42	252	40890	4256	109	163	387	30.14	183	24494	9424	146	130	346	29.68	143	20618	8573	155
	Uniform	495	498	49.28	646	106683	10778	110	843	1079	31.16	1187	104592	27122	138	468	685	41.9	569	72757	17356	163
8	Gamma	481	467	45.76	644	103628	10120	110	883	1097	33.28	1313	112384	28077	146	470	669	39.5	581	74410	17168	163
	Log-normal	453	471	44.34	616	97404	10181	110	919	1180	31.64	1296	116186	30739	132	444	675	37.48	563	70843	17370	163

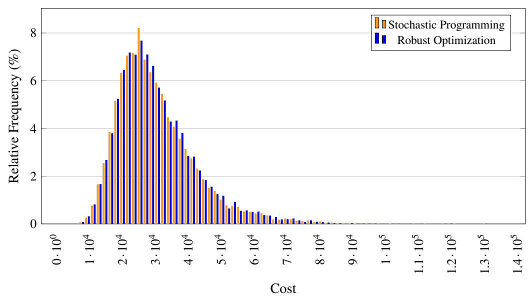
Table 4.9 – Comparison of solutions characteristics focused on minimum worst-case.



(a) Uniform demand distribution.

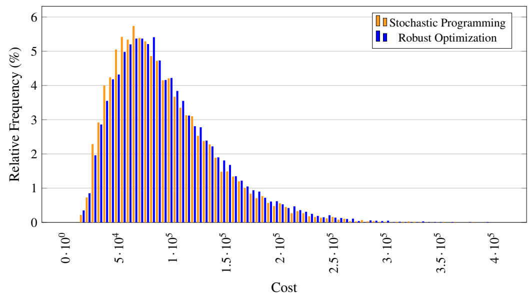


(b) Gamma demand distribution.

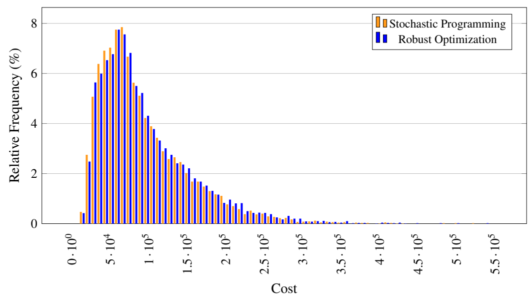


(c) Log-normal demand distribution.

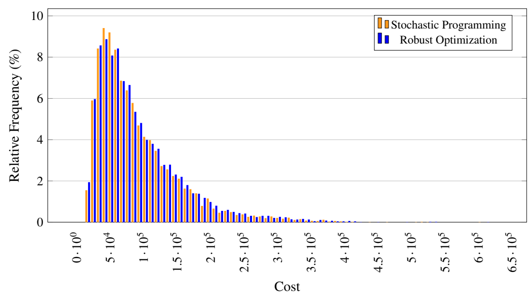
Figure 4.15 – Robust optimization and stochastic programming cost relative frequency for the instance class 2.



(a) Uniform demand distribution.



(b) Gamma demand distribution.



(c) Log-normal demand distribution.

Figure 4.16 – Robust optimization and stochastic programming cost relative frequency for the instance class 8.

CHAPTER 5

Integrating lot-sizing and scheduling under multistage demand uncertainty

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Adaptation and Approximate Strategies for Solving the Lot-sizing and Scheduling Problem Under Multi-stage Demand Uncertainty

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Abstract

This work addresses the lot-sizing and scheduling problem under multistage demand uncertainty. A flexible production system is considered, with the possibility to adjust the size and the schedule of lots in every time period based on a rolling-horizon planning scheme. Computationally intractable multistage stochastic programming models are often employed on this problem. An adaptation strategy to the multistage setting for two-stage programming and robust optimization models is proposed. We also present an approximation heuristic strategy to address the problem more efficiently, relying on multistage stochastic programming and adjustable robust optimization. In order to evaluate each strategy and model proposed, a Monte Carlo simulation experiment under a rolling-horizon scheme is performed. Results show that the strategies are promising in solving large-scale problems: approximate strategy based on adjustable robust optimization has, on average, 6.72% better performance and is 7.9 times faster than the deterministic model.

Keywords Lot-sizing and Scheduling Problem; GLSP; Adjustable Robust Optimization; Multistage Stochastic Programming; Rolling-horizon

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5.1. Introduction

The integration of lot-sizing and scheduling problems has been widely addressed in the literature and its benefits and applications are also shown in many previous contributions (Maravelias and Sung, 2009; Copil et al., 2016; Almada-Lobo et al., 2015). Moreover, significant improvements on the formulation of lot-sizing and scheduling models have been achieved in recent works (Guimaraes et al., 2014), which has substantially increased the efficiency of solving complex and practical production planning problems. Nevertheless, as most of the works consider deterministic parameters (Hu and Hu, 2016), the literature on lot-sizing and scheduling under uncertainty is scarce, especially regarding multistage decision-making.

In this work, we are considering the integration of lot-sizing and scheduling decisions under multistage uncertain demand. Several modeling approaches can be used to address demand uncertainty in lot-sizing and scheduling problems. Two straightforward exact approaches would be formulating the problem as a multistage stochastic programming model or using a deterministic model embedded into a rolling-horizon planning scheme. Nevertheless, these approaches have particular issues: multistage stochastic programming models are usually intractable and deterministic models may produce inferior solutions in uncertain settings.

Therefore, the main idea of this paper is to provide high quality solutions in short computational runtimes for the lot-sizing and scheduling problem under multistage demand uncertainty by means of the following solving strategies: i) Adaptating the two-stage stochastic programming and robust optimization models to a multistage structure; and ii) Approximating heuristically the multistage stochastic programming and adjustable robust optimization models. We also introduce two traditional approaches for the problem as a comparison basis to the strategies proposed: iii) Deterministic model with safety stocks incorporated; and iv) Classical multistage stochastic programming model. We believe that it is the first time the integration of lot-sizing and scheduling under uncertainty is tackled via the strategies proposed. Moreover, as far as we know, adjustable robust optimization has never been applied in lot-sizing and scheduling problems.

The proposed strategies seem to be promising in reducing the computational complexity of lot-sizing and scheduling problems under uncertainty, especially when there is a high number of products or a long planning horizon. The adaptation strategy allows the application of static robust optimization and two-stage stochastic programming models for solving problems under multistage uncertainty through a rolling-horizon planning scheme. Usually, these models are more tractable than the multistage stochastic programming, for instance, the static robust optimization presents a similar tractability of its deterministic version. The approximate strategy reduces the complexity of the problem by eliminating the setup (binary) variables for future periods, which allows to improve the tractability of the multistage stochastic programming and adjustable robust optimization models without much loss of their solution quality.

We consider the General Lot-sizing and Scheduling Problem (GLSP) as the base formulation for the models proposed. In addition, the strategies presented are assessed via a Monte Carlo simulation experiment under a rolling-horizon planning scheme and are evaluated in terms of average cost, standard deviation, worst-case scenario cost, best-case scenario cost and runtime. At the end, we draw the main advantages and drawbacks of each strategy and provide guidelines based on the experiment results.

Rolling-horizon planning scheme is generally used in a stochastic environment (Sethi and Sorger, 1991) or for reducing computational times when solving detailed industrial problems with long planning horizons. For instance, De Araujo et al. (2007); Clark and Clark (2000); Li and Ierapetritou (2010) propose rolling-horizon schemes to reduce the computational requirements to solve complex lot-sizing and scheduling problems. Using stochastic programming in a rolling-horizon environment, Beraldi et al. (2008), Wu and Ierapetritou (2007) and Sand and Engell (2004) address uncertainty in lot-sizing and scheduling problems. In the robust optimization field, Bredström et al. (2013) propose a rolling-horizon method to tackle demand uncertainty in production planning. Based on the previous works, we present a rolling-horizon planning scheme in the Section 5.3, it will be a common setting in the Monte Carlo simulation for each model proposed.

The paper is organized as follows. Section 5.2 presents the related works on lot-sizing and scheduling problems under uncertainty. Section 5.3 states the problem, introduces the rolling-horizon planning scheme, describes the deterministic GLSP model with safety stocks and presents the exact multistage stochastic programming model. Section 5.4 introduces the multistage adaptation strategy for the two-stage stochastic programming and robust optimization models. Section 5.5 proposes an approximate heuristic strategy to achieve better computational performance for multistage stochastic programming and adjustable robust optimization models. In Section 5.6, the Monte Carlo simulation experiment is described, its results are discussed and guidelines are provided. Finally, Section 5.7 concludes this work and provides further research directions.

5.2. Related works

In the past years, many of the contributions considering uncertainty in lot-sizing and scheduling relied on stochastic programming, hierarchical production planning schemes (Wu and Ierapetritou, 2007), meta-heuristics (Ramezanian and Saidi-Mehrabad, 2013) and rolling-horizon planning strategies (Beraldi et al., 2008). More recently, robust optimization models have been used to incorporate uncertainty in lot-sizing and scheduling problems (Gabrel et al., 2014; Alem et al., 2016). The main contributions to the uncertainty field in lot-sizing and scheduling problems are presented next.

To overcome the computational complexity of solving an integrated monolithic model, some authors resort to hierarchical production planning schemes within optimization frameworks. For instance, Meybodi and Foote (1995) address production planning and scheduling under demand and production failure using a stochastic hierarchical production planning model with scheduling and rolling-horizon heuristics. Sand and Engell (2004) develop a two-stage stochastic programming approach with a Lagrangian decomposition in order to solve the hierarchical scheduling of flexible chemical batch processes with uncertainty in capacity and demand. Wu and Ierapetritou (2007) also address the production planning and scheduling problem under uncertainty through a hierarchical approach with a rolling-horizon strategy, however the authors develop a multistage stochastic model to tackle demand uncertainty.

Some efforts were devoted to address uncertainty in lot-sizing and scheduling problems using stochastic programming and to improve its intractability. In order to increase the solving efficiency of batching and scheduling problems under demand uncertainty, Balasubramanian and Grossmann (2004) propose an approximation strategy to the multistage stochastic optimization, which is achieved by embedding two-stage stochastic models into a shrinking-horizon solution scheme. Ramezanian and Saidi-Mehrabad (2013) use metaheuristics to tackle the lot-sizing and scheduling problem considering multistage production system and parallel machines under uncertain processing times and product demand. Also considering parallel machines, Beraldi et al. (2006) develop a multistage stochastic programming and apply a fix-and-relax heuristic to solve a lot-sizing and scheduling problem with sequence-dependent set-up costs and uncertain processing times. In a subsequent work, Beraldi et al. (2008) focus on identical parallel machines, providing rolling-horizon and fix-and-relax heuristics, which can be applied in large-scale applications, such as in the fiberglass and textile industries. More recently and targeting the automotive industry, Hu and Hu (2016) propose a two-stage stochastic model and use a scenario reduction approach to deal with demand uncertainty in the lot-sizing and scheduling problem.

Notwithstanding the scarcity of works integrating lot-sizing and scheduling under uncertainty, there is a wide range of works that address both problems independently under uncertainty using either stochastic programming or robust optimization approaches. For instance, literature reviews in lot-sizing (Aloulou et al., 2014) or lot-scheduling problems (Sox et al., 1999) present several works that incorporate uncertainty using methods, such as stochastic, fuzzy models and simulation. The majority of these studies only consider a single uncertainty modeling technique and do not benchmark performance against with other uncertainty modeling approaches.

With the recent advances in the field of robust optimization (Bertsimas and Sim, 2004; Ben-Tal et al., 2004; Gabrel et al., 2014), some works have started applying these techniques to lot-sizing and scheduling and related problems. For instance, Chunpeng and Gang (2009) develop a hierarchical optimization-simulation approach for the production planning and scheduling in refineries. The authors incorporate uncertain demand using a robust optimization model and resort to a rolling-horizon planning scheme that includes the detailed simulation scheduling only for the current period. Alem et al. (2016) develop a robust optimization model for the general lot-sizing and scheduling problem and compare it with a two-stage stochastic model using Monte Carlo simulation. Contributions using robust optimization have also been addressed in related fields, such as inventory management (Klabjan et al., 2013; Zhang, 2011; See and Sim, 2010; Ben-Tal et al., 2004; Qiu and Shang, 2014), inventory routing (Solyali et al., 2012), lot-sizing and cutting (Alem and Morabito, 2012), production planning (Bredström et al., 2013) and others (Gabrel et al., 2014).

Significant advances in adjustable robust optimization (ARO) have been made on lot-sizing and production planning problems. Zhang et al. (2016) develop an adjustable robust mixed-integer linear programming model to solve the scheduling of continuous industrial processes with interruptible load under uncertainty. Lappas and Gounaris (2016) also apply ARO for process scheduling under uncertainty in processing times, showing that the ARO model achieves superior performance than the static robust optimization version. Using ARO in lot-sizing problems, Postek and Den Hertog propose a methodology to solve the model that splits the uncertainty sets and constructs decision rules for integer and continuous variables. Melamed et al. (2016) develop a linear model using affinely adjustable robust optimization to address demand uncertainty in the single-product production planning problem. The model accounts for backlog and inventory costs, however it does not have production capacity neither setup constraints. To the best of our knowledge, ARO approaches have not yet been applied in the integration of lot-sizing and scheduling problems under demand uncertainty.

Acknowledging on one hand the scarcity of works looking at multistage demand uncertainty in the lot-sizing and scheduling problem, and on the other hand the intractability of the majority of the respective approaches, different strategies are proposed in this work in order to efficiently tackle demand uncertainty in a multistage decision setting and rolling-horizon planning scheme.

5.3. Problem statement and traditional modeling approaches

We consider a multiproduct, multiperiod and capacitated lot-sizing and scheduling problem under independent and dynamic demand uncertainty. Changeovers incur in dependent setup times and costs and backlogs are allowed. The objective is to attend the demand at minimum cost, which includes setup, inventory and backlog costs. A flexible production system is present, where production, scheduling and inventory decisions can be adjusted in every period. Moreover, within a rolling-horizon planning scheme, all decisions are taken for the current period with: 1) the past decisions fixed; 2) the demand of current period revealed; and 3) the mean and the coefficient of variation of future demand known.

The next subsections are organized in the following way. We first present the rolling-horizon planning scheme that will be used in the Monte Carlo simulation for each model proposed. Secondly, we formally describe the deterministic formulation for the lot-sizing and scheduling problem. Next, we show how safety stocks can be incorporated into the deterministic model and then present the exact multistage stochastic programming model for the GLSP.

5.3.1 Rolling-horizon planning scheme

The rolling-horizon planning scheme can be divided into two parts. The first refers to the current period, for which the demand information is known and inventory, lot-sizing and scheduling decisions are not to be taken right away. The second part is related to the future periods where the demand is unknown and the decisions should not be immediately implemented. After the decisions of the current period have been implemented, the following period becomes the current period, the respective demand is revealed and decisions are then implemented. The iterative approach continues until all decisions within the planning horizon are taken. Figure 5.1 illustrates how the rolling-horizon scheme can be applied in a setting with demand uncertainty. We intend to evaluate and compare the solutions of all the models proposed within this rolling-horizon planning scheme.

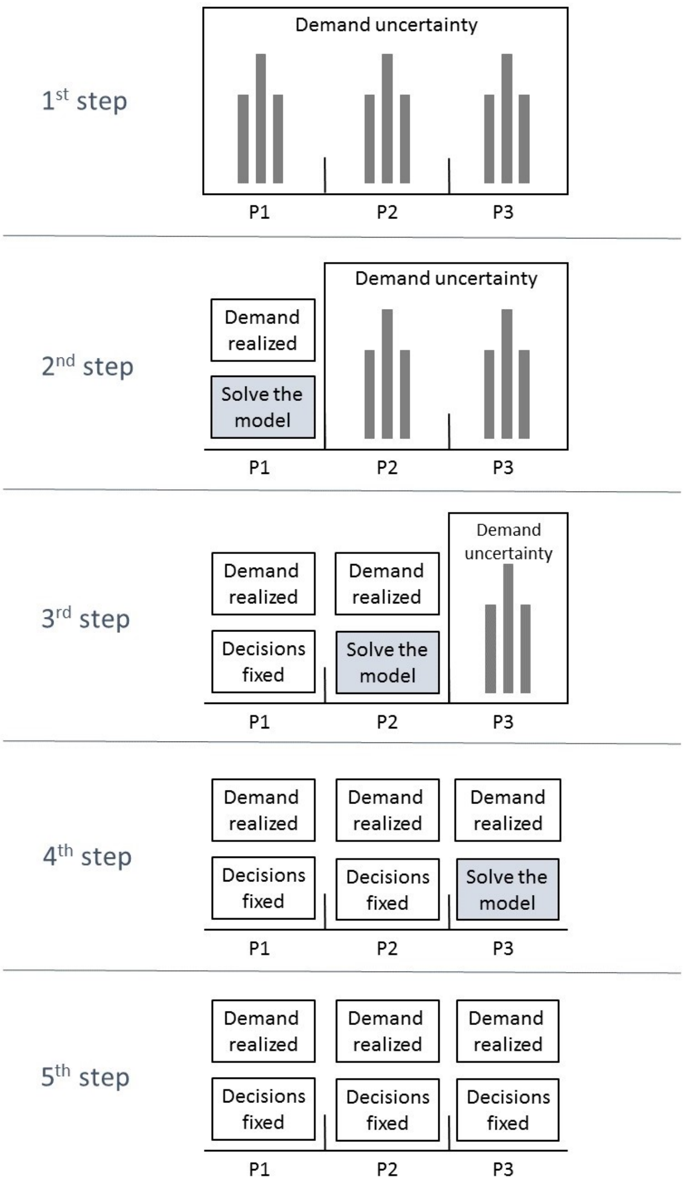

Figure 5.1 – Rolling-horizon planning scheme example for 3 time periods.

5.3.2 Deterministic standard approach

The classical GLSP is one of the most used formulations for solving deterministic lot-sizing and scheduling problems. The formulation is as follows (Fleischmann and Meyr, 1997; Meyr, 2002):

(F1: `DetModel`)

$$\min \sum_{j \in J} \sum_{t \in T} \left(h_j^+ \cdot I_{jt}^+ + h_j^- \cdot I_{jt}^- \right) + \sum_{j,\ell \in J} \sum_{n \in N} s_{j\ell} \cdot Z_{j\ell n} \tag{5.1}$$

$$\text{s.t.: } I_{j(t-1)}^+ + I_{jt}^- + \sum_{n \in N_t} X_{jn} = I_{jt}^+ + I_{j(t-1)}^- + d_{jt}, \ \forall j \in J, \ t \in T \tag{5.2}$$

$$\sum_{j \in J} \sum_{n \in N_t} p_j \cdot X_{jn} + \sum_{j,\ell \in J} \sum_{n \in N_t} q_{j\ell} \cdot Z_{j\ell n} \leq cap_t, \ \forall t \in T \tag{5.3}$$

$$X_{jn} \leq b_{jt} \cdot Y_{jn}, \ \forall j \in J, \ t \in T, \ n \in N_t \tag{5.4}$$

$$\sum_{j \in J} Y_{jn} = 1, \ \forall n \in N \tag{5.5}$$

$$\sum_{\ell \in J} Z_{j\ell n} = Y_{j(n-1)}, \ \forall j \in J, \ n \in N \tag{5.6}$$

$$\sum_{j \in J} Z_{j\ell n} = Y_{\ell n}, \ \forall \ell \in J, \ n \in N \tag{5.7}$$

$$X_{jn} \geq m_j \cdot \left(Y_{jn} - Y_{j(n-1)} \right), \ \forall j \in J, \ n \in N \tag{5.8}$$

$$Y_{jn} \in \mathbb{B}, \ X_{jn}, \ Z_{j\ell n}, \ I_{jt}^+, \ I_{jt}^- \geq 0, \ \forall j, \ell \in J, \ n \in N, \ t \in T. \tag{5.9}$$

The parameter J refers to the set of products and T the set of periods. N_t is the subset of micro-periods of period t, such that $\bigcup_{t \in T} N_t = N$. The parameters h_j^+, h_j^-, $s_{j\ell}$, d_{jt}, p_j, $q_{j\ell}$, cap_t, b_{jt} and m_j, stand for holding cost, shortage cost, sequence-dependent setup cost, demand, production time, setup time, capacity, maximum lot and minimum lot-sizes, respectively. Decision variables X_{jt}, I_{jt}^+, I_{jt}^-, Y_{jn} and $Z_{j\ell n}$ are related to production, inventory, backlogging, setup and changeover among two products, respectively.

Objective function (5.1) minimizes the total cost. Constraints (5.2) are the inventory balance constraints and (5.3) the production capacity. Constraints (5.4) allow the production of product j only if its setup occurs in the respective micro-period. Constraints (5.5) limits to one the number of setups in each micro-period. Constraints (5.6) and (5.7) enforce the relation of setup and changeover states. To ensure the triangular inequality, Constraints (5.8) establish a minimum lot size for product j in case it was not produced in the last micro-period. Constraints (5.9) define the variables domain.

5.3.3 Safety stock approach

Many authors (Bredström et al., 2013; Rafiei et al., 2015; Absi and Kedad-Sidhoum, 2009) incorporate safety stocks in production planning optimization approaches to tackle product demand variability. Absi and Kedad-Sidhoum (2009) treat the safety stock level as one of the objectives in the lot-sizing problem, rather than a constraint, which maintains the feasibility for every safety stock level desired. Using as a basis the work of Absi and Kedad-Sidhoum (2009), the `DetModel` can be reformulated to incorporate safety stocks in the GLSP as follows:

(F2: `SafetyDetModel`)

$$\min \sum_{j\in J}\sum_{t\in T}\left(sc_j^+ \cdot S_{jt}^+ + sc_j^- \cdot S_{jt}^- + h_j^- \cdot I_{jt}^-\right) + \sum_{j,\ell\in J}\sum_{n\in N} s_{j\ell}\cdot Z_{j\ell n} \tag{5.10}$$

s.t.: Constraints (5.3), (5.4), (5.5), (5.6), (5.7), (5.8)

$$S_{j(t-1)}^+ + S_{jt}^- + I_{jt}^- + \sum_{n\in N_t} X_{jn} =$$

$$\delta_{jt} + S_{jt}^+ + S_{j(t-1)}^- + I_{j(t-1)}^- + d_{jt}, \ \forall j\in J,\ t\in T \tag{5.11}$$

$$S_{jt}^- \le sl_{jt}, \ \forall j\in J,\ t\in T \tag{5.12}$$

$$Y_{jn}\in\mathbb{B},\ X_{jn},\ Z_{j\ell n},\ S_{jt}^+, S_{jt}^-,\ I_{jt}^- \ge 0,\ \forall j,\ell\in J,\ n\in N,\ t\in T. \tag{5.13}$$

Note that variables I_{jt}^+ are now replaced by variables S_{jt}^+ and S_{jt}^-, that stand for overstock and safety stock deficit, respectively. sc_j^+ and sc_j^- are penalties for overstock and safety stock deficit, respectively. Clearly, sc_j^- has to be lower than h_j^-. Parameter sl_{jt} represents the desired safety stock level for period t and product j. δ_{jt} is the safety stock variation in period t, i.e. $\delta_{jt} = sl_{jt} - sl_{j(t-1)}$. Constraints (5.11) are equivalent to Constraints (5.2) and Constraints (5.12) limit the maximum safety stock deficit to the safety stock level. Objective function (5.10) now minimizes backlog, setup costs, overstock and safety stock deficit.

5.3.4 Exact multistage stochastic programming approach

In the multistage stochastic programming model version of the GLSP, uncertainty is modeled through a finite number of scenarios $|K|$, and each scenario k has a probability π_k of demand realization and the summation of all probabilities must be 1, i.e. $\sum_k \pi_k = 1$. It is assumed that production quantities and production sequences can be readjusted in every time period, as well as inventory and demand fulfilment decisions. A model based on a scenario tree (Brandimarte, 2006), in which scenarios and periods are represented by a set of nodes that specify the potential realization of the uncertainty, is described below:

(F3: `MstageStochModel`)

$$\min \sum_{m\in M}\pi_m\cdot\left(\sum_{j\in J}\left(h_j^+\cdot I_{jt_m m}^+ + h_j^-\cdot I_{jt_m m}^-\right) + \sum_{j,\ell\in J}\sum_{n\in N_m} s_{j\ell}\cdot Z_{j\ell nm}\right) \tag{5.14}$$

$$\text{s.t.: } I_{jt_a a}^+ + I_{jt_m m}^- + \sum_{n\in N_m} X_{jnm} = I_{jtn}^+ + I_{jt_a a}^- + d_{jt_m m},\ \forall j\in J,\ m\in M, a = a_m \tag{5.15}$$

$$\sum_{j\in J}\sum_{n\in N_m} p_j\cdot X_{jnm} + \sum_{j,\ell\in J}\sum_{n\in N_m} q_{j\ell}\cdot Z_{j\ell nm} \le cap_{t_m},\ \forall m\in M \tag{5.16}$$

$$X_{jnm} \le b_{jt}\cdot Y_{jnm},\ \forall j\in J,\ m\in M,\ n\in N_m \tag{5.17}$$

$$\sum_{j\in J} Y_{jnm} = 1,\ \forall m\in M,\ n\in N_m \tag{5.18}$$

$$\sum_{\ell \in J} Z_{j\ell nm} = Y_{j(n-1)m'}, \; \forall j \in J, \; m \in M, \; m' \in \{m\} \cup \{a_m\}, \; n \in N_m \tag{5.19}$$

$$\sum_{j \in J} Z_{j\ell nm} = Y_{\ell nm}, \; \forall \ell \in J, \; m \in M, \; n \in N_m \tag{5.20}$$

$$X_{jnm} \geq m_j \cdot \left(Y_{jnm} - Y_{j(n-1)m'}\right), \; \forall j \in J, \; m \in M, \; m' \in \{m\} \cup \{a_m\}, \; n \in N_m \tag{5.21}$$

$$Y_{jnm} \in \mathbb{B}, \; X_{jnm}, \; Z_{j\ell m}, \; I^+_{jtm}, \; I^-_{jtm} \geq 0, \; \forall j, \ell \in J, \; \forall m \in M, \; \forall n \in N, \tag{5.22}$$

where, M is the set of nodes, N_m is the set that contains the micro-periods related to node m, a_m is the immediate predecessor of node m and t_m is the period of node m. In the model, production and scheduling decisions, as well as inventory and demand fulfilment variables are adjusted in every period (for each demand realization), which relates to a more flexible production system.

5.4. Adapting uncertainty models to the multistage setting

In this section we first present a two-stage stochastic programming and a robust optimization model to solve the GLSP under demand uncertainty. Then we apply the shrinking-horizon approach (Balasubramanian and Grossmann, 2004) as a strategy to adapt both models to the multistage structure.

5.4.1 Two-stage stochastic programming model

The following two-stage stochastic programming model has production and setup as here-and-now variables, while inventory and demand fulfillment are considered wait-and-see variables. Its objective function aims at minimizing the expected cost and Constraints (5.24) represent the inventory balance for each scenario $k \in K$:

(F4: `2stageStochModel`)

$$\min \sum_{j \in J} \sum_{t \in T} \sum_{k \in K} \pi_k \cdot \left(h^+_j \cdot I^+_{jtk} + h^-_j \cdot I^-_{jtk}\right) + \sum_{j,\ell \in J} \sum_{n \in N} s_{j\ell} \cdot Z_{j\ell n} \tag{5.23}$$

s.t.: Constraints (5.3), (5.4), (5.5), (5.6), (5.7), (5.8)

$$I^+_{j(t-1)k} + I^-_{jtk} + \sum_{n \in N_t} X_{jn} = I^+_{jtk} + I^-_{j(t-1)k} + d_{jtk}, \; \forall j \in J, \; t \in T, \; k \in K \tag{5.24}$$

$$Y_{jn} \in \mathbb{B}, \; X_{jn}, \; Z_{j\ell n}, \; I^+_{jtk}, \; I^-_{jtk} \geq 0, \; \forall j, \ell \in J, \; n \in N, \; t \in T, \; k \in K. \tag{5.25}$$

5.4.2 Robust optimization model

In the robust optimization modeling approach, uncertainty is modeled using an interval-polyhedral uncertainty set. Following the budget of uncertainty approach proposed by Bertsimas and Sim (2004), the demand uncertainty is modeled using the polyhedral-uncertainty set U:

$$U = \left\{ \mathbf{D} \in \mathbb{R}_{+}^{|J| \times |T|} | \; \xi_{jt} \in [-1,1], \; \sum_{\tau=1}^{t} |\xi_{j\tau}^{d}| \leq \Gamma_{jt}, \; \forall j \in J, \; t \in T \right\}, \tag{5.26}$$

in which $\xi_{jt} = \left(\tilde{d}_{jt} - d_{jt}\right) / \hat{d}_{jt}$ is the scaled demand deviation and $\tilde{d}_{jt}$ is the bounded random demand variable in the interval $[d_{jt} - \hat{d}_{jt}, d_{jt} + \hat{d}_{jt}]$. The budget of uncertainty parameter Γ_{jt}^{d} reflects risk preferences and controls the maximum number of coefficients that can assume extreme values.

To determine the tractable robust counterpart of the GLSP with demand uncertainty, the balancing Constraints (5.2) need to be replaced by inequalities constraints. Otherwise, those constraints make the robust formulation generally intractable (Melamed et al., 2016). Therefore, Constraints (5.2) are replaced by (see Melamed et al. (2016); Alem et al. (2016) for more details):

$$H_{jt} \geq h_{jt}^{+} \cdot I_{jt} = h_{jt}^{+} \cdot \left[I_{j0}^{+} + \sum_{\tau=1}^{t} \sum_{n \in N_{\tau}} X_{jn} - \sum_{\tau=1}^{t} d_{j\tau} \right], \; \forall j \in J, \; t \in T, \tag{5.27}$$

and

$$H_{jt} \geq h_{jt}^{-} \cdot (-I_{jt}) = h_{jt}^{-} \cdot \left[I_{j0}^{-} - \sum_{\tau=1}^{t} \sum_{n \in N_{\tau}} X_{jn} + \sum_{\tau=1}^{t} d_{j\tau} \right], \; \forall j \in J, \; t \in T. \tag{5.28}$$

Using the uncertainty set U, the worst-case realization of the demand is now introduced into the new Constraints (5.29) and (5.30):

$$H_{jt} \geq h_{jt}^{+} \cdot I_{jt} = h_{jt}^{+} \cdot \left[I_{j0}^{+} + \sum_{\tau=1}^{t} \sum_{n \in N_{\tau}} X_{jn} - \min_{\tilde{\mathbf{d}} \in U} \sum_{\tau=1}^{t} (d_{j\tau} + \hat{d}_{j\tau} \cdot \xi_{j\tau}^{d}) \right], \; \forall j \in J, \; t \in T, \tag{5.29}$$

and

$$H_{jt} \geq h_{jt}^{-} \cdot (-I_{jt}) = h_{jt}^{-} \cdot \left[I_{j0}^{-} - \sum_{\tau=1}^{t} \sum_{n \in N_{\tau}} X_{jn} + \max_{\tilde{\mathbf{d}} \in U} \sum_{\tau=1}^{t} (d_{j\tau} + \hat{d}_{j\tau} \cdot \xi_{j\tau}^{d}) \right], \; \forall j \in J, \; t \in T. \tag{5.30}$$

By applying transformation techniques (Bertsimas and Sim, 2004) used in robust optimization (see Appendix for the detailed derivation), the robust optimization problem is reformulated into its tractable finite counterpart:

(F5: `RobModel`)

$$\min \sum_{j\in J}\sum_{t\in T} H_{jt} + \sum_{j,\ell\in J}\sum_{n\in N} s_{j\ell}\cdot Z_{j\ell n} \tag{5.31}$$

s.t.: Constraints (5.3), (5.4), (5.5), (5.6), (5.7), (5.8)

$$H_{jt} \geq h^{+}_{jt}\cdot\left(I^{+}_{0t} + \sum_{\tau=1}^{t}\sum_{n\in N_\tau} X_{jn} - \sum_{\tau=1}^{t} d_{j\tau} + \Gamma^{d}_{jt}\cdot\lambda^{d}_{jt} + \sum_{\tau=1}^{t}\mu^{d}_{j\tau t}\right),\ \forall j\in J,\ t\in T \tag{5.32}$$

$$H_{jt} \geq h^{-}_{jt}\cdot\left(I^{-}_{0t} - \sum_{\tau=1}^{t}\sum_{n\in N_\tau} X_{jn} + \sum_{\tau=1}^{t} d_{j\tau} + \Gamma^{d}_{jt}\cdot\lambda^{d}_{jt} + \sum_{\tau=1}^{t}\mu^{d}_{j\tau t}\right),\ \forall j\in J,\ t\in T \tag{5.33}$$

$$\lambda^{d}_{jt} + \mu^{d}_{j\tau t} \geq \hat{d}_{j\tau},\ \forall j\in J,\ t\in T,\ \tau\leq t \tag{5.34}$$

$$\lambda^{d}_{jt},\ \mu^{d}_{j\tau t} \geq 0,\ ,\ \forall j\in J,\ t\in T,\ \tau\leq t \tag{5.35}$$

$$Y_{jn}\in\mathbb{B},\ X_{jn},\ Z_{j\ell n}\geq 0,\ \forall j,\ell\in J,\ n\in N,\ t\in T. \tag{5.36}$$

5.4.3 Shrinking-horizon approach as an adaptation to the multistage setting

In both two-stage stochastic programming and robust optimization models, the here-and-now decisions are related to production quantities and setup scheduling. Usually, these models can be applied in rigid production planning systems, in which it is not possible to easily readjust the production planning and scheduling sequences in every time period. However, if a flexible production system is considered, in which it is possible to adjust the production quantities and scheduling decisions every period, those models may be ineffective.

Focusing on the latter case, it is possible to adapt formulations F4 and F5 to the multistage setting. Balasubramanian and Grossmann (2004) propose an approach called shrinking-horizon, which uses the two-stage programming model as an approximation model for the multistage problem. In this method, the standard two-stage stochastic model is formulated for the entire planning horizon T. In the first iteration ($i = 1$), the model is solved and then the variables from the first period (stage) are fixed. After that, the model is solved again for the next period with the remaining periods $|T| - i$ and variables from previous periods fixed. The algorithm proceeds until the decision variables from all periods are fixed and then the cost is computed. The pseudo-algorithm can be described as follows:

Algorithm 2: Shrinking-horizon heuristic

t(current time period) = 0
TH(planning horizon) = $|T|$
while $t \leq TH$ **do**
  0) Demand realization for period t
  1) Solve `2stageStochModel` or `RobModel` for t to TH
  2) Fix decision variables X, Z, Y I^- and I^+ for period t
  if $t = TH$ **then**
    3) Compute total cost
  4) Update $t = t + 1$

The main difference between shrinking-horizon heuristic and the rolling-horizon planning scheme is that the planning horizon considered is fixed for the shrinking-horizon heuristic, which is not always true for the rolling-horizon planning scheme. Since we are considering a fixed planning horizon for the Monte Carlo simulation, both approaches can be considered similar. Hence, we apply the shrinking-horizon heuristic in both robust and stochastic models of this section as a more computationally efficient alternative to the exact multistage stochastic model embedded in the rolling-horizon planning.

5.5. Approximate heuristic strategy for rolling-horizon planning

The rolling-horizon heuristic with approximate models proposed by Clark and Clark (2000) gives a reliable approximation of the regular rolling-horizon scheme for lot-sizing with sequence-dependent setups. Moreover, it is considerably faster than the standard rolling-horizon scheme. The idea behind this method is to solve the model in a rolling-horizon environment, but relaxing the integer setup variables that are not within the current period. After the model is solved, the current period is updated and the setup variables of the previous period are fixed, then the model is solved again with the setup variables of the next periods relaxed.

The main argument of using approximation heuristics (De Araujo et al., 2007) is that there is no need for detailed schedules for later periods in rolling-horizon planning because decisions for later periods are never implemented in the current period. Hence, it is possible to use a simplified version that is easier to solve for future periods. This simplified version already used in deterministic lot-sizing and scheduling problems can be extended to uncertainty models, such as multistage stochastic programming and adjustable robust optimization models, in order to overcome their intractability.

5.5.1 Approximate multistage stochastic programming model

In this work, we first adapt this heuristic for the multistage stochastic model. In each period (stage) the model is solved with the corresponding setup variables being integer, with fixed variables from previous periods, and relaxed setup variables for future periods. Hence, the reformulated model will have $|J|\times|N_t|$ integer variables, instead of $\sum_t^{|T|}(|J|\times|N_t|\times\prod_{t'}^{t}|K_{t'}|)$ from the regular model, $|K_{t'}|$ being the number of scenarios in period t'. Therefore, the approximate multistage stochastic programming model is formulated as follows:

(F6: `AppMstageStochModel`)

$$\min \sum_{m\in M}\pi_m\cdot\left(\sum_{j\in J}\left(h_j^+\cdot I^+_{jt_mm}+h_j^-\cdot I^-_{jt_mm}\right)\right)+\sum_{m\in MP}\pi_m\cdot\left(\sum_{j,\ell\in J}\sum_{n\in N_m}s_{j\ell}\cdot Z_{j\ell nm}\right)$$
$$+\sum_{m\in MF}\pi_m\cdot\left(\sum_{j\in J}\sum_{n\in N_m}c_j\cdot X_{jnm}\right) \tag{5.37}$$

$$\text{s.t.: } I^+_{jt_aa}+I^-_{jt_mm}+\sum_{n\in N_m}X_{jnm}=I^+_{jtn}+I^-_{jt_aa}+d_{jt_mm},\ \forall j\in J,\ m\in M \tag{5.38}$$

$$\sum_{j\in J}\sum_{n\in N_m} p_j \cdot X_{jnm} + \sum_{j,\ell\in J}\sum_{n\in N_m} q_{j\ell}\cdot Z_{j\ell nm} \leq cap_{t_m},\ \forall m\in MP \tag{5.39}$$

$$\sum_{j\in J}\sum_{n\in N_m} u_j \cdot X_{jnm} \leq cap_{t_m},\ \forall m\in MF \tag{5.40}$$

$$X_{jnm} \leq b_{jt}\cdot Y_{jnm},\ \forall j\in J,\ m\in MP,\ n\in N_m \tag{5.41}$$

$$\sum_{j\in J} Y_{\ell nm} = 1,\ \forall m\in MP,\ n\in N_m \tag{5.42}$$

$$\sum_{\ell\in J} Z_{j\ell nm} = Y_{j(n-1)m'},\ \forall j\in J,\ m\in MP,\ m'\in\{m\}\cup\{a_m\},\ n\in N_m \tag{5.43}$$

$$\sum_{j\in J} Z_{j\ell nm} = Y_{jnm},\ \forall \ell\in J,\ m\in MP,\ n\in N_m \tag{5.44}$$

$$X_{jnm} \geq m_j\cdot\left(Y_{jnm} - Y_{j(n-1)m'}\right),\ \forall j\in J,\ m\in MP,\ m'\in m\cup a_m,\ n\in N_m \tag{5.45}$$

$$X_{jnm},\ Z_{j\ell m},\ I^{+}_{jtm},\ I^{-}_{jtm} \geq 0,\ \forall j,\ell\in J,\ \forall m\in M,\ \forall n\in N \tag{5.46}$$

$$Y_{jnm}\in\mathbb{B},\ \forall j\in J,\ \forall m\in MP,\ \forall n\in N \tag{5.47}$$

Where MP is the set related to the past and current periods and MF is the set that contains nodes related to future periods. In Constraints (5.40), the production time of future periods is increased using u_j factor, to compensate for the relaxed setup time. u_j is the approximated production rate for product j. Usually this parameter is calculated using the demand mean ($dmean_j$) of product j over periods and the mean setup time to product j, (q_j^{-to}):

$$u_j = (q_j^{-to} + p_j\cdot dmean_j)/dmean_j. \tag{5.48}$$

In the objective function (5.37), the setup cost for future periods is replaced by c_j, which is the approximated setup cost for product j per unit produced. Usually, it is calculated taking into account the demand mean ($dmean_j$) of product j over periods and its mean setup cost to product j (s_j^{-to}):

$$c_j = s_j^{-to}/dmean_j. \tag{5.49}$$

One problem with this traditional estimation is that it can be rather conservative when discrepancies between setup time and costs among products are high. Therefore, we propose to follow a weighted mean approach to fine-tune the parameters u_j and c_j:

Algorithm 3: Fine tuning method for parameters u_j and c_j

1) Solve `DetModel` for a specific instance

2.1) $q_j^{-to} = \sum_{\ell \in J} \sum_{n \in N} q_{\ell j} \cdot Z_{\ell jn} / |N|$

2.2) $u_j = (q_j^{-to} + p_j \cdot dmean_j) / dmean_j$

3.1) $s_j^{-to} = \sum_{\ell \in J} \sum_{n \in N} s_{\ell j} \cdot Z_{\ell jn} / |N|$

3.2) $c_j = s_j^{-to} / dmean_j$

With this weighted mean approach, only the setup times and costs that were executed in the determinist solution will be taken into account when calculating parameters u_j and c_j. This fine tuning avoids calculating a mean that takes into account setups that may never occur, especially when the variability of dependent setup times and costs among products is high.

5.5.2 Approximate affinely adjustable robust optimization model

One of the assumptions of the robust optimization (RO) approach is that all the variables are here-and-now decisions. As mentioned before, this assumption may be unrealistic for multistage optimization problems or may produce highly conservative solutions. Therefore, we propose an adjustable robust optimization formulation for the GLSP. In adjustable robust models, it is possible to have wait-and-see variables, i.e., some decisions can be adjusted after the uncertainty is revealed. The main drawback of ARO models is their intractability, as shown in Ben-Tal et al. (2004). Nevertheless, it is possible to approximate the continuous wait-and-see variables using affine functions in order to keep tractability of the model. For instance, the production decision variables X_{jn} and the auxiliary variable H_{jt} would be replaced by the following variables (see Melamed et al. (2016) for more details), respectively:

$$X_{jn}(d_{jt}) = X_{jn}^0 + \sum_{r=1}^{t-1} X_{jn}^r \cdot d_{jr}, \ \forall j \in J, \ t \in T, \ n \in N_t, \tag{5.50}$$

$$H_{jt}(d_{jt}) = H_{jt}^0 + \sum_{r=1}^{t-1} H_{jt}^r \cdot d_{jr}, \ \forall j \in J, \ t \in T. \tag{5.51}$$

Bertsimas et al. (2010) proved that this approximation provides optimal solutions when uncertainty is modeled using box sets. However, for mixed-integer linear programming models, it is not possible to model integer variables as wait-and-see decisions using parametric decision rules. In this case, it is necessary to resort to others approaches. Bertsimas and Caramanis (2010) introduce an approach that splits an uncertainty set Z into smaller I subsets Z_i that have their own integer variables. The drawback here is that the resulting model will have I more integer variables, increasing its computational complexity. Postek and Den Hertog propose a similar method for splitting the uncertainty sets and constructing decision rules for integer and continuous variables. However, these approaches are still highly intractable for large problems (Gorissen et al., 2015).

To deal with integer variables in the GLSP, we propose to apply the same approximation strategy (see Section 5.5.1) on the ARO model within the rolling-horizon perspective. Hence, the ARO model contains the setup integer variables only for the current period, which are considered here-and-now variables, and continuous setup variables for future periods. For this purpose, we divided the planning periods of the model in present period (TP) and future periods (TF). Hence, by applying the approximate techniques (described in Section 5.5.1) and the affine functions (Constraints (5.50) and (5.51)) for the future variables, the approximate ARO model can be modeled as follows:

(F7a: `AppAROModel`)

$$\min \sum_{j\in J}\sum_{t\in TP} H'_{jt} + \sum_{j,\ell\in J}\sum_{t\in TP}\sum_{n\in N_t} s_{j\ell}\cdot Z_{j\ell n}$$

$$+ \sum_{t\in TF}\sum_{j\in J}(c_j\cdot(\sum_{n\in N_t} X^0_{jn} + \sum_{r=|TP|+1}^{t-1} X^r_{jn}\cdot d_{j\tau}) + H^0_{jt} + \sum_{r=|TP|+1}^{t-1} H^r_{jt}\cdot d_{jr}) \tag{5.52}$$

s.t.:

$$H'_{jt} \geq h^+_{jt}\cdot\left(I^+_{0t} + \sum_{\tau=1}^{t}(\sum_{n\in N_\tau} X'_{jn} - d_{j\tau})\right),\ \forall j\in J,\ t\in TP \tag{5.53}$$

$$H'_{jt} \geq h^-_{jt}\cdot\left(I^-_{0t} + \sum_{\tau=1}^{t}(\sum_{n\in N_\tau} -X'_{jn} + d_{j\tau})\right),\ \forall j\in J,\ t\in TP \tag{5.54}$$

$$\begin{aligned} 0 \geq & -(H^0_{jt} + \sum_{r=|TP|+1}^{t-1} H^r_{jt}\cdot d_{jr}) \\ & + h^+_{jt}\cdot\left(I^+_{0t} + \sum_{\tau=|TP|+1}^{t}(\sum_{n\in N_\tau}(X^0_{jn} + \sum_{r=|TP|+1}^{\tau-1} X^r_{jn}\cdot d_{jr}) - d_{j\tau})\right. \\ & \left. + \sum_{\tau\in TP}(\sum_{n\in N_\tau} X'_{jn} - d_{j\tau})\right),\ \forall j\in J,\ t\in TF \end{aligned} \tag{5.55}$$

$$\begin{aligned} 0 \geq & -(H^0_{jt} + \sum_{r=|TP|+1}^{t-1} H^r_{jt}\cdot d_{jr}) \\ & + h^-_{jt}\cdot\left(I^-_{0t} - \sum_{\tau=|TP|+1}^{t}(\sum_{n\in N_\tau} -(X^0_{jn} + \sum_{r=|TP|+1}^{\tau-1} X^r_{jn}\cdot d_{jr}) + d_{j\tau})\right. \\ & \left. + \sum_{\tau\in TP}(\sum_{n\in N_\tau} -X'_{jn} + d_{j\tau})\right),\ \forall j\in J,\ t\in TF \end{aligned} \tag{5.56}$$

$$\sum_{j\in J}\sum_{n\in N_t} p_j\cdot X'_{jn} + \sum_{j,\ell\in J}\sum_{n\in N_t} q_{j\ell}\cdot Z_{j\ell n} \leq cap_t,\ \forall t\in TP \tag{5.57}$$

$$\sum_{j\in J} u_j \cdot \sum_{n\in N_t} (X^0_{jn} + \sum_{r=1}^{t-1} X^r_{jn} \cdot d_{jr}) \le cap_t,\ \forall t \in TF \tag{5.58}$$

$$X'_{jn} \le b_{jt} \cdot Y_{jn},\ \forall j \in J,\ t \in TP,\ n \in N_t \tag{5.59}$$

$$\sum_{j\in J} Y_{jn} = 1,\ \forall t \in TP,\ n \in N \tag{5.60}$$

$$\sum_{\ell\in J} Z_{j\ell n} = Y_{j(n-1)},\ \forall t \in TP,\ j \in J,\ n \in N_t \tag{5.61}$$

$$\sum_{j\in J} Z_{j\ell n} = Y_{\ell n},\ \forall \ell \in J,\ t \in TP,\ n \in N_t \tag{5.62}$$

$$X'_{jn} \ge m_j \cdot \left(Y_{jn} - Y_{j(n-1)}\right),\ \forall j \in J,\ t \in TP, \in N_t \tag{5.63}$$

$$X^0_{jn} + \sum_{r=|TP|+1}^{t-1} X^r_{jn} \cdot d_{jr} \ge 0,\ \forall j \in J,\ t \in TF,\ n \in N_t \tag{5.64}$$

$$H^0_{jt} + \sum_{r=|TP|+1}^{t-1} H^r_{jt} \cdot d_{jr} \ge 0,\ \forall j \in J,\ t \in TF \tag{5.65}$$

$$Y_{jn} \in \mathbb{B},\ X'_{jn},\ Z_{j\ell n} \ge 0,\ \forall j,\ell \in J,\ n \in N,\ t \in T. \tag{5.66}$$

Where, variables X'_{jn} and H'_{jt} refer to production and inventory/backlog costs in the current periods, respectively. Objective function (5.52) contains the approximation of future setup costs and minimizes the setup and inventory/backlog costs related to the current period and future periods. Constraints (5.53) and (5.54) are the inventory and backlog constraints for the current period. Constraints (5.55) and (5.56) are the inventory and backlog balance for future periods. Constraints (5.57) and (5.58) limit the production capacity for the current and future periods, respectively. Constraints (5.59) -(5.63) enforce the sequence-dependent setup setting and are equivalent to Constraints (5.4)-(5.8), but now they are applied only for the current period. Constraints (5.64) and (5.65) define the domain of variables for future periods. Constraints (5.66) define the domain of variables for the current period.

By using the uncertainty set U and applying robust optimization transformations techniques (see Appendix), we can obtain the following solvable mixed-integer linear model:

(F7c: `AppAROModel`)

$$\min \sum_{j\in J}\sum_{t\in TP} H_{jt} + \sum_{j,\ell\in J}\sum_{t\in TP}\sum_{n\in N_t} s_{j\ell} \cdot Z_{j\ell n} + \sum_{t\in TF}\sum_{j\in J}(\sum_{n\in N_t} c_j \cdot X^0_{jn} + H^0_{jt})$$
$$+ \sum_{t\in TF}\sum_{j\in J}(\sum_{r=|TP|+1}^{t-1} (\sum_{n\in N_t} X^r_{jn} \cdot c_j + H^r_{jt}) \cdot d_{jr} + \Gamma_{jt-1} \cdot \lambda^A_{jt}$$
$$+ \sum_{\tau=|TP|+1}^{t-1} \mu^A_{j\tau t} + \Gamma_{jt-1} \cdot \lambda^F_{jt} + \sum_{\tau=|TP|+1}^{t-1} \mu^F_{j\tau t}) \tag{5.67}$$

s.t.:

$$H_{jt} \geq h^{+}_{jt} \cdot \left(I^{+}_{0t} + \sum_{\tau=1}^{t} \big(\sum_{n \in N_\tau} X'_{jn} - d_{j\tau} \big) \right), \ \forall j \in J,\ t \in TP \tag{5.68}$$

$$H_{jt} \geq h^{-}_{jt} \cdot \left(I^{-}_{0t} + \sum_{\tau=|TP|+1}^{t} \big(\sum_{n \in N_\tau} -X'_{jn} + d_{j\tau} \big) \right), \ \forall j \in J,\ t \in TP \tag{5.69}$$

$$\begin{aligned} 0 \geq h^{+}_{jt} \cdot \Bigg(& I^{+}_{0t} + \sum_{\tau=|TP|+1}^{t} \sum_{n \in N_\tau} X^{0}_{jn} + \sum_{\tau=|TP|+1}^{t} \big(\sum_{n \in N_\tau} \sum_{r=|TP|+1}^{\tau-1} X^{r}_{jn} \cdot d_{jr} - d_{j\tau} \big) \\ & + \sum_{\tau \in TP} \big(\sum_{n \in N_\tau} X'_{jn} - d_{j\tau} \big) \Bigg) \\ & - \big(H^{0}_{jt} + \sum_{r=|TP|+1}^{t-1} H^{r}_{jt}\big) + \Gamma_{jt} \cdot \lambda^{B}_{jt} + \sum_{\tau=|TP|+1}^{t} \mu^{B}_{j\tau t}, \forall j \in J,\ t \in TF \end{aligned} \tag{5.70}$$

$$\begin{aligned} 0 \geq h^{-}_{jt} \cdot \Bigg(& I^{-}_{0t} - \sum_{\tau=|TP|+1}^{t} \sum_{n \in N_\tau} X^{0}_{jn} + \sum_{\tau=|TP|+1}^{t} \big(\sum_{r=|TP|+1}^{\tau-1} \sum_{n \in N_\tau} -X^{r}_{jn} \cdot d_{jr} + d_{j\tau} \big) \\ & + \sum_{\tau \in TP} \big(\sum_{n \in N_\tau} -X'_{jn} + d_{j\tau} \big) \Bigg) \\ & - \big(H^{0}_{jt} + \sum_{r=|TP|+1}^{t-1} H^{r}_{jt}\big) + \Gamma_{jt} \cdot \lambda^{C}_{jt} + \sum_{\tau=|TP|+1}^{t} \mu^{C}_{j\tau t}, \ \forall j \in J,\ t \in TF \end{aligned} \tag{5.71}$$

$$\sum_{j \in J} \sum_{n \in N_t} p_j \cdot X'_{jn} + \sum_{j,\ell \in J} \sum_{n \in N_t} q_{j\ell} \cdot Z_{j\ell n} \leq cap_t, \ \forall t \in TP \tag{5.72}$$

$$\begin{aligned} & \sum_{j \in J} \sum_{n \in N_t} u_j \cdot X^{0}_{jn} + \sum_{j \in J} \sum_{n \in N_t} u_j \cdot \sum_{r=|TP|+1}^{t-1} X^{r}_{jn} \cdot d_{jr} \\ & + \sum_{j \in J} \big(\Gamma_{jt-1} \cdot \lambda^{D}_{jt} + \sum_{\tau=|TP|+1}^{t-1} \mu^{D}_{j\tau t}\big) \leq cap_t, \ \forall t \in TF \end{aligned} \tag{5.73}$$

$$X^{0}_{jn} + \sum_{r=|TP|+1}^{t-1} X^{r}_{jn} \cdot d_{jr} - \big(\Gamma_{jt-1} \cdot \lambda^{E}_{jn} + \sum_{\tau=|TP|+1}^{t-1} \mu^{E}_{j\tau n}\big) \geq 0, \ \forall j \in J,\ t \in TF,\ n \in N_t, \tag{5.74}$$

$$H^{0}_{jt} + \sum_{r=|TP|+1}^{t-1} H^{r}_{jt} \cdot d_{jr} - \big(\Gamma_{jt-1} \cdot \lambda^{F}_{jt} + \sum_{\tau=|TP|+1}^{t-1} \mu^{F}_{j\tau t}\big) \geq 0, \ \forall j \in J,\ t \in TF, \tag{5.75}$$

$$X'_{jn} \leq b_{jt} \cdot Y_{jn}, \ \forall j \in J,\ t \in TP,\ n \in N_t \tag{5.76}$$

$$\sum_{j \in J} Y_{jn} = 1, \ \forall t \in TP,\ n \in N \tag{5.77}$$

$$\sum_{\ell \in J} Z_{j\ell n} = Y_{j(n-1)}, \ \forall t \in TP,\ j \in J,\ n \in N_t \tag{5.78}$$

$$\sum_{j \in J} Z_{j\ell n} = Y_{jn}, \ \forall \ell \in J,\ t \in TP,\ n \in N_t \tag{5.79}$$

$$X'_{jn} \geq m_j \cdot \big(Y_{jn} - Y_{j(n-1)}\big), \ \forall j \in J,\ t \in TP, \in N_t \tag{5.80}$$

$$\lambda^A_{jt} + \mu^A_{j\tau t} \geq \sum_{n \in N_t} c_j \cdot X^\tau_{jn} \cdot \hat{d}_{j\tau}, \ \forall j \in J, \ t \in TF, \ \tau < t \tag{5.81}$$

$$\lambda^A_{jt}, \ \mu^A_{j\tau t} \geq 0, \ \forall j \in J, \ t \in TF, \ \tau < t \tag{5.82}$$

$$\mu^{B1}_{j\tau t} + \mu^{B2}_{j\tau t} + \nu^{B1}_{j\tau t} + \nu^{B2}_{j\tau t} = (\sum_{\tau=r+1}^{t} \sum_{n \in N_\tau} X^r_{jn} \cdot h^+_{jt} - H^r_{jt} - h^+_{jt}) \cdot \hat{d}_{j\tau}, \ \forall j \in J, \ t \in TF, \ \tau < t \tag{5.83}$$

$$\mu^{B1}_{j\tau t} + \mu^{B2}_{j\tau t} + \nu^{B1}_{j\tau t} + \nu^{B2}_{j\tau t} = -\hat{d}_{j\tau} \cdot h^+_{jt}, \ \forall j \in J, \ t \in TF, \ \tau = t \tag{5.84}$$

$$\lambda^B_{jt} - \nu^{B1}_{j\tau t} + \nu^{B2}_{j\tau t} \geq 0, \ \forall j \in J, \ t \in TF, \ \tau \leq t \tag{5.85}$$

$$\lambda^B_{jt}, \ \mu^{B1}_{j\tau t}, \ \nu^{B1}_{j\tau t} \geq 0, \ \forall j \in J, \ t \in TF, \ \tau \leq t \tag{5.86}$$

$$\mu^{B2}_{j\tau t}, \ \nu^{B2}_{j\tau t} \leq 0, \ \forall j \in J, \ t \in TF, \ \tau \leq t \tag{5.87}$$

$$\mu^{C1}_{j\tau t} + \mu^{C2}_{j\tau t} + \nu^{C1}_{j\tau t} + \nu^{C2}_{j\tau t} = (h^-_{jt} - \sum_{\tau=r+1}^{t} \sum_{n \in N_\tau} X^r_{jn} \cdot h^-_{jt} - H^r_{jt}) \cdot \hat{d}_{j\tau}, \ \forall j \in J, \ t \in TF, \ \tau < t \tag{5.88}$$

$$\mu^{C1}_{j\tau t} + \mu^{C2}_{j\tau t} + \nu^{C1}_{j\tau t} + \nu^{C2}_{j\tau t} = \hat{d}_{j\tau} \cdot h^-_{jt}, \ \forall j \in J, \ t \in TF, \ \tau = t \tag{5.89}$$

$$\lambda^C_{jt} - \nu^{C1}_{j\tau t} + \nu^{C2}_{j\tau t} \geq 0, \ \forall j \in J, \ t \in TF, \ \tau \leq t \tag{5.90}$$

$$\lambda^C_{jt}, \ \mu^{C1}_{j\tau t}, \ \nu^{C1}_{j\tau t} \geq 0, \ \forall j \in J, \ t \in TF, \ \tau \leq t \tag{5.91}$$

$$\mu^{C2}_{j\tau t}, \ \nu^{C2}_{j\tau t} \leq 0, \ \forall j \in J, \ t \in TF, \ \tau \leq t \tag{5.92}$$

$$\lambda^D_{jt} + \mu^D_{j\tau t} \geq \sum_{n \in N_t} u_j \cdot X^\tau_{jn} \cdot \hat{d}_{j\tau}, \ \forall j \in J, \ t \in TF, \ \tau < t \tag{5.93}$$

$$\lambda^D_{jt}, \ \mu^D_{j\tau t} \geq 0, \ \forall j \in J, \ t \in TF, \ \tau < t \tag{5.94}$$

$$\lambda^E_{jn} + \mu^E_{j\tau n} \geq X^\tau_{jn} \cdot \hat{d}_{j\tau}, \ \forall j \in J, \ t \in TF, \ \tau < t \tag{5.95}$$

$$\lambda^E_{jt}, \ \mu^E_{j\tau t} \geq 0, \ \forall j \in J, \ t \in TF, \ \tau < t \tag{5.96}$$

$$\lambda^F_{jt} + \mu^F_{j\tau t} \geq H^\tau_{jt} \cdot \hat{d}_{j\tau}, \ \forall j \in J, \ t \in TF, \ \tau < t \tag{5.97}$$

$$\lambda^F_{jt}, \ \mu^F_{j\tau t} \geq 0, \ \forall j \in J, \ t \in TF, \ \tau < t \tag{5.98}$$

$$Y_{jn} \in \mathbb{B}, \ X'_{jn}, \ Z_{j\ell n} \geq 0, \ \forall j, \ell \in J, \ n \in N, \ t \in T. \tag{5.99}$$

5.6. Computational experiment

With this computational experiment we intend to assess each strategy and model proposed mainly in terms of average costs and under four other criteria: standard deviation, worst-case scenario cost, best-case scenario cost and runtime. With these criteria it is possible to evaluate the trade-offs of the proposed strategies regarding quality of solutions and computational performance, and then provide guidelines of the strategies that solve the problem in the most practicable manner. The experiment is implemented in C++ programming language and the models were solved using CPLEX 12.6 optimization tool on an Intel E5-2450 processor with a Scientific Linux 6.5 platform.

This section is divided into 3 parts. Subsection 5.6.1 explains the Monte Carlo simulation experiment and how the performance of the proposed strategies and models can be evaluated with this procedure. In the second subsection, we list the sizes and structures of the instance sets generated. Finally, Subsection 5.6.3 presents the results of the experiment with the main remarks and guidelines.

5.6.1 Monte Carlo simulation as evaluation method

The experiment is carried out in the following way for all the proposed models. First, the models are solved with the demand of the current period revealed and only the mean and coefficient of variation of future demand known. After the model is solved, the decisions of the current period are fixed, the demand of the next period is revealed and the model is solved again. The method continues until the demand for all periods is revealed and the decisions for all periods are fixed. Thus, after repeating the simulation a significant number of times, it is possible to calculate the performance of each model in terms of average cost, standard deviation, worst and best-case scenario cost and runtime.

We perform fifty replications in order to achieve statistically significant results without compromising the computational complexity of the experiment. Moreover, we use the same seed generator for all the simulations. The demand distribution is assumed to be log-normal, also the mean and variability values are given for the respective product and period. As mentioned before, the product demand is independent for every product j and time period t. Furthermore, we assume that the time limit to solve the model in each period is 10 minutes.

In order to achieve the best potential of each model in terms of reducing the average cost, we test several combinations of uncertainty parameters for each approach. For the ARO (RO) models, the experiment is performed with the variability level $\hat{\gamma}$ varying from 0 to 3 (0.5 to 2.5) with a step of 0.1, in which $\hat{d}_{j\tau} = \hat{\gamma} \cdot \sigma$ and σ is the standard deviation. Also, we assume four different budgets of uncertainty (Γ_{jt}): t, $0.5t+0.5$, $0.1t+0.5$ and $0.05t+0.1$, where t is the time period. For the stochastic models, we consider 5 different numbers of scenarios: 10, 50, 100, 200, and 500. The scenarios were generated using the Sample-Average Approximation technique (Kleywegt et al., 2002). For the deterministic model with safety stock, the desired safety stock level sl_{jt} was varied from 0 to 0.5, with a step of 0.01. Therefore, besides comparing the models, the simulation method is used to evaluate the impact of the combination of the uncertainty parameters. This allows for selecting the combination of parameters that provides the best performance for each modeling approach.

We also evaluated the static strategies in the simulation for the following models: deterministic, deterministic with safety stocks, robust optimization and two-stage stochastic programming with 10 scenarios. This analysis is important to quantify the value of using a rolling-horizon planning scheme.

5.6.2 Instances

We organize our instances in 2 classes by changing the production capacity:

1. High capacity: $cap_t = \frac{\sum_{j \in J} \mathbf{E}(d_{jt})}{0.75}$;

2. Low capacity: $cap_t = \frac{\sum_{j \in J} \mathbf{E}(d_{jt})}{0.90}$,

where $\sum_{j \in J} \mathbf{E}(d_{jt})$ is the sum of the expected demand for all products in period t.

For each class, we create two sets of instances with different instance sizes: small (with $|P| = 3$ and $|T| = 3$) and medium ($|P| = 5$ and $|T| = 4$). Each set contains 5 instances. The number of micro-periods per period of each instance is equal to the number of products (i.e., $|N_t| = |P|$). Also, the parameters of each instance are generated based on Amorim et al. (2013) and according to the following structure:

1. Expected demand: $\mathbf{E}(d_{jt}) = U(120, 480)$;
2. Coefficient of variation: 0.5;
3. Setup time (same product type): $q_{j=\ell} = U(1, 10)$;
4. Setup time (different product type): $q_{j \neq \ell} = U(11, 50)$;
5. Holding cost: $h_j^+ = U(1, 10)$;
6. Shortage cost: $h_j^- = 10 \cdot h_j^+$;
7. Setup cost among same products type: $s_{j=\ell} = 0$;
8. Setup cost among different products type (for medium instances): $s_{j \neq \ell} = \frac{(h_j^- + h_j^+)}{2} \cdot 5 \cdot U(0, 1)$;
9. Setup cost among different products type (for small instances): $s_{j \neq \ell} = \frac{(h_j^- + h_j^+)}{2} \cdot 10 \cdot U(0, 1)$;
10. Production time: $p_j = 1$;
11. Minimum lot size: $m_j = b_{jt}$;
12. Maximum lot size: $m_j = cap_t$.

With this arrangement we can examine the performance of each strategy as a function of the production capacity, number of products and length of the planning horizon.

5.6.3 Overall results

Tables 5.1 and 5.2 present the performance of each strategy proposed for instances with low and high production capacities, respectively. As mentioned before, among all uncertainty parameters combinations tested for each strategy, we selected the one that has the best performance in terms of average cost. The heading of the tables has the following structure: the strategy and modeling technique proposed, then the average cost, standard deviation, worst-case cost, best-case cost and runtime for small and medium instances. Also, the tables are divided in 3 different categories: the first category contains the static approaches without rolling-horizon planning, the second indicates the expected value given perfect information, finally the strategies within the rolling-horizon planning scheme are presented.

	Low production capacity	Small (3 products and 3 periods)					Medium (5 products and 4 periods)				
	Strategy	Average cost	Standard deviation	Worst-case cost	Best-case cost	Runtime (s)	Average cost	Standard deviation	Worst-case cost	Best-case cost	Runtime (s)
Static	Deterministic	42,966	35,721	198,310	4,454	0.06	121,210	74,898	390,729	13,084	5.48
	Deterministic w/ safety stocks	40,521	32,975	180,056	4,691	0.06	106,385	63,495	361,200	19,914	4.07
	Two-stage stochastic programming	39,079	30,541	168,888	4,710	0.18	99,176	55,935	310,976	21,565	10.86
	Robust optimization	37,806	30,009	167,680	5,178	0.13	98,698	59,106	353,325	21,538	22.73
	Expected value given perfect information	12,448	17,774	84,245	234	0.04	11,752	17,434	85,491	442	6.50
Rolling-horizon planning scheme	Deterministic	14,324	19,694	106,953	234	5.08	15,675	17,358	94,940	442	470.39
	Deterministic w/ safety stocks	14,324	19,694	106,953	234	5.26	15,675	17,358	94,940	442	483.96
	Exact multistage stochastic programming	13,862	19,399	105,729	848	31,844.16	14,317	17,646	96,479	1,107	56,089.02
	Adapted two-stage stochastic programming	13,842	19,451	105,729	773	10.58	13,909	17,414	94,940	1,589	16,345.18
	Adapted robust optimization	13,741	19,519	106,919	982	7.81	14,098	17,553	94,940	1,360	2,053.67
	Approximate multistage stochastic programming	13,909	19,448	105,730	676	35	14,215	17,488	94,940	1,152	552.44
	Approximate adjustable robust optimization	13,756	19,507	105,729	850	12.95	14,082	17,565	94,940	1,477	48.66
	Approximate multistage stochastic programming (tuned)	13,870	19,384	105,734	590	133.84	14,372	17,272	94,940	895	2,923.69
	Approximate adjustable robust optimization (tuned)	13,729	19,491	105,729	895	11.18	14,138	17,500	94,940	1,556	57.12

Table 5.1 – Performance of the strategies proposed for instances with low capacity.

High production capacity		Small (3 products and 3 periods)					Medium (5 products and 4 periods)				
	Strategy	Average cost	Standard deviation	Worst-case cost	Best-case cost	Runtime (s)	Average cost	Standard deviation	Worst-case cost	Best-case cost	Runtime (s)
Static	Deterministic	41,391	34,042	189,144	4,846	0.06	129,533	77,318	381,926	17,261	7.27
	Deterministic w/ safety stocks	33,281	27,352	155,449	5,538	0.10	97,547	60,877	339,580	22,852	2.19
	Two-stage stochastic programming	27,009	20,106	133,039	4,181	0.13	81,668	49,769	318,684	23,883	5.99
	Robust optimization	24,376	18,891	124,731	6,027	0.09	72,417	48,131	290,816	22,528	23.44
	Expected value given perfect information	3,080	5,675	41,734	207	0.03	1,908	2,870	17,871	313	7.47
Rolling-horizon planning scheme	Deterministic	4,060	7,552	54,561	207	3.57	3,964	6,470	37,408	313	457.55
	Deterministic w/ safety stocks	4,060	7,552	54,561	207	3.92	3,964	6,470	37,408	313	429.19
	Exact multistage stochastic programming	3,883	6,776	51,026	379	31923.76	3,999	6,281	37,408	313	47568.46
	Adapted two-stage stochastic programming	3,981	6,434	51,026	532	32.14	4,921	3,516	20,219	1,395	11576.59
	Adapted robust optimization	3,812	6,766	51,026	304	5.63	3,826	5,895	33,181	678	782.46
	Approximate multistage stochastic programming	3,982	6,515	51,558	616	859.84	3,891	4,926	25,951	698	3516.19
	Approximate adjustable robust optimization	3,859	6,634	51,558	295	9.37	3,808	5,462	32,159	714	49.24
	Approximate multistage stochastic programming (tuned)	3,941	6,523	48,271	352	551.00	3,848	5,966	37,344	540	15970.60
	Approximate adjustable robust optimization (tuned)	3,844	6,775	51,558	247	10.67	3,755	4,894	24,980	581	39.61

Table 5.2 – Performance of the strategies proposed for instances with high capacity.

Table 5.3 describes the overall performance of each strategy using a rolling-horizon planning scheme compared to the deterministic rolling-horizon planning scheme. The first column displays the strategies proposed, the second column shows how better each strategy proposed is when compared to the deterministic approach, and the last column shows the ratio of the deterministic runtime over the proposed strategy runtime.

Firstly, it is important to notice that the simulation experiment confirms some intuitive thoughts regarding uncertainty in production planning. The value of perfect information (VPI)* decreases when the production capacity is more restrictive and the value of stochastic solution (VSS)† is lower for high production capacity. The value of using a rolling-horizon planning scheme (VRHS)‡, which is the difference between the static and rolling-horizon solution, also decreases if there is low production capacity, it means that to take full advantage of flexible decisions and rolling-horizon planning schemes it is important to have high production capacity.

Results also suggest that the performance of each strategy is more affected by the advantages and drawbacks of the modeling approach and by the size of instances (number of products and planning horizon) rather than the production capacity of each instance. It is also important to notice the high disparities of the simulation runtime, especially for the stochastic models. This happens because sometimes models with a small number of scenarios have a better performance in terms of average cost than the ones with a large number of scenarios.

Below we analyze the performance of each strategy and provide some insights regarding solution quality and computational complexity. The strategies are organized in the following way: standard strategies (deterministic, deterministic with safety stocks and exact multistage stochastic programming models within the rolling-horizon planning scheme), adaptation strategies (two-stage stochastic programming and robust optimization models adapted to the multistage setting using the shrinking-horizon planning scheme) and approximate strategies (approximate multistage stochastic programming and approximate adjustable robust optimization models in the rolling-horizon planning scheme with and without fine tuning).

Standard strategies: The deterministic model in the rolling-horizon planning scheme is one of the fastest strategies. It is 178 times faster than the multistage stochastic programming model. However, it assumes only the mean as information of future demand, which results in inferior here-and-now solutions and a general poor performance, when compared with other methods. The incorporation of safety stocks does not help to reduce the average cost, however it can reduce the worst-case scenario cost. Hence, the incorporation of safety stocks can be considered a conservative robust approach, somehow similar to the Soyster's model (Soyster, 1973). Because of that, we argue that using robust optimization models that incorporate uncertainty using polyhedral uncertainty sets may bring a preciser

*VPI average value for low capacity instances: 14.12%. VPI average value for high capacity instances: 36.72%.

†VSS average value for low capacity instances: 6.07%. VSS average value for high capacity instances: 1.77%.

‡VRHS average value for low capacity instances: 81.73%. VRHS average value for high capacity instances: 95.31%.

Strategy	(Deterministic-Strategy)/Deterministic performance	Deterministic/Strategy runtime
Deterministic	0%	1.00
Deterministic w/ safety stocks	0%	1.015
Exact multistage stochastic programming	5.16%	0.006
Adapted two-stage stochastic programming	3.60%	0.033
Adapted robust optimization	6.70%	0.329
Approximate multistage stochastic programming	5.32%	0.189
Approximate multistage stochastic programming (tuned)	5.24%	0.048
Approximate adjustable robust optimization	6.62%	7.790
Approximate adjustable robust optimization (tuned)	6.72%	7.899

Table 5.3 – Overall performance of each strategy using a rolling-horizon planning scheme in the simulation experiment.

trade-off than safety stocks models in terms of average cost and risk.

The exact multistage stochastic programming performance is, on average, 5.16% better than the deterministic model, but it is 166.6 times slower. When compared with other strategies it does not achieve a good performance for medium instances because of its intractability. The model is not able to converge within the 10 minutes limit for each period, even for a small number of scenarios, resulting in high gaps and poor solution quality.

Adaptation strategies: The two-stage stochastic programming adapted to the multistage setting has, on average, solution with 3.60% better quality than the deterministic model. It achieves the best solution for large instances with low capacity, however it can take a significant amount of time to be solved, on average it requires 6 times more than the deterministic model. Still, this adaptation strategy is more tractable than the traditional multistage stochastic programming, as also shown by Balasubramanian and Grossmann (2004). One possible drawback of this strategy is that we are considering production and setup decision as first-stage variables, even for future periods, which is conservative and may increase the inventory costs. An alternative approach would be considering the future production and scheduling decisions as wait-and-see decisions (with the risk of being under conservative for instances with low production capacity).

The robust optimization has the second best overall performance in the simulation experiment. On average its performance is 6.70% better than the deterministic model, with similar computational complexity (3 times slower). The good performance of this approach may be caused by the following aspects: the rolling-horizon planning scheme eliminates the static decisions from the robust optimization models and allows for dynamic production and scheduling decisions; also, the unnecessity of scenarios to incorporate uncertainty keeps the tractability of the model reasonable. However, this approach may require to test several combinations of variability levels and budget of uncertainty profiles to obtain a setting that is neither under nor over-conservative. An alternative to testing several robust optimization parameters is to obtain a reasonable budget of uncertainty using the probability of violating the demand constraints (see Wei et al. (2011) for more details) or choosing a budget of uncertainty based on previous experiences from similar instances (see Alem et al. (2016) for more details).

Approximation strategies: On average, the approximate multistage stochastic programming has almost the same performance (5.32% better than the deterministic model) as the exact multistage stochastic programming model. It is 0.82% worse for small instances and 1.15% better for large instances than the exact one, suggesting that this strategy may be more suitable for long planning horizons. Generally, it is 33 times faster than the exact multistage stochastic programming model, however it still requires generating scenarios, which increases the intractability and may demand a high quantity of scenarios in order to provide good solutions.

The approximate adjustable robust optimization (tuned) has the best overall performance (6.72% better and 7.9 times faster than the deterministic model). We believe that there are two main reasons for the high efficiency of this strategy. The first one is that the relaxation strategy considers the scheduling integer decisions only for the current period, which substantially reduces the number of binary variables in the mixed-integer program model. The second is that the robust optimization model does not requires scenario generation to incorporate uncertainty, which makes it more tractable. These aspects allow to tackle longer planning horizons or even higher number of products. However, it has the same disadvantage of the robust model - the need for testing several uncertainty parameters combinations in order to generate good solutions.

The tuning of the approximation parameters slightly contributed to improve the performances only for the approximate adjustable robust model. The tune of the approximate parameters c_j and u_j improved the solutions by 0.11 p.p. on average.

Given the high efficiency of the approximate adjustable robust optimization, we present in Table 5.4 the results of the simulation experiment for large instances (7 products and 8 periods) to show the practicability of the approximate adjustable robust optimization in solving large instances of the problem.

Overall, the strategies that employ robust optimization approaches achieve better results: approximate adjustable robust optimization and adapted robust optimization have similar solution quality, but the first strategy is considerably faster than the second. Strategies that employ stochastic programming models also achieve acceptable results. The approximate multistage stochastic programming model achieve a slightly better performance than the exact multistage stochastic programming model and is almost 8 times faster. The solutions provided by the adapted two-stage stochastic programming is considerably worst than the exact multistage stochastic programming model, but it is still a faster option than the standard method.

5.7. Conclusion

This work presents adaptation and approximation strategies for improving the solving efficiency of the GLSP under demand uncertainty in a multistage setting. These strategies are alternatives to the multistage stochastic programming and the standard rolling horizon approaches. The main premise to apply these strategies is that in a flexible production system, the solutions implemented are only here-and-now decisions, hence future decision variables can be approximated in order to improve solving efficiency without much loss of solution quality. A simulation experiment was developed to compare the strategies proposed with the main standard approaches: deterministic GLSP, deterministic GLSP with safety stocks and multistage stochastic programming models within rolling-horizon planning schemes. The simulation results indicate that the strategies proposed are suitable to solve the problem, especially for long planning horizons or large quantity of final products. The approximate adjustable robust optimization is able to provide rapid and high quality solutions for any of the tested class/size of instances and is the only method that is able to provide adequate solutions for large planning horizons in the simulation experiment.

Besides the promising results, we believe that there is still room for improvement. Future work lies on the study of machine learning to create more precise approximations and/or rules to efficiently provide near-optimum decisions in the rolling-horizon setting. A study that compares these strategies with meta-heuristics and other heuristics (e.g., hierar-

Low production capacity	Large (7 products and 8 periods)				
Strategy	Average cost	Standard deviation	Worst-case cost	Best-case cost	Runtime (s)
Deterministic	13,329	12,040	57,517	2,049	13,771.02
Approximate adjustable robust optimization (tuned)	9,412	7,701	53,880	3,584	673.34

Table 5.4 – Performance of the deterministic model and the approximate adjustable robust optimization strategy for large instances.

chical approaches, rolling-horizon heuristics) in a uncertainty setting can be very valuable. Opportunities also lie on applying and evaluating these strategies in other lot-sizing settings such as multi-level lot-sizing, lot-sizing with parallel machines or with uncertain processing/setup times.

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Appendix 5.A Robust optimization transformations

The worst-case realization of the demand uncertainty is finally achieved by solving the nonlinear new constraints (5.100) and (5.101) over the uncertainty set, i.e.:

$$H_{jt} \geq h^{+}_{jt} \cdot I_{jt} = h^{+}_{jt} \cdot \left[I^{+}_{j0} + \sum_{\tau=1}^{t} \sum_{n \in N_\tau} X_{jn} - \min_{\tilde{\mathbf{d}} \in U} \sum_{\tau=1}^{t} (d_{j\tau} + \hat{d}_{j\tau} \cdot \xi^{d}_{j\tau}) \right], \ \forall j \in J,\ t \in T, \tag{5.100}$$

and

$$H_{jt} \geq h^{-}_{jt} \cdot (-I_{jt}) = h^{-}_{jt} \cdot \left[I^{-}_{j0} - \sum_{\tau=1}^{t} \sum_{n \in N_\tau} X_{jn} + \max_{\tilde{\mathbf{d}} \in U} \sum_{\tau=1}^{t} (d_{j\tau} + \hat{d}_{j\tau} \cdot \xi^{d}_{j\tau}) \right], \ \forall j \in J,\ t \in T. \tag{5.101}$$

Both inner optimization problems in constraints (5.100) and (5.101) lead to the following auxiliary primal (dual) problem for each pair (j,t):

$$\begin{aligned} \max \quad & \sum_{\tau=1}^{t} \hat{d}_{j\tau} \cdot \xi^{d}_{j\tau} \\ \text{s.t.:} \quad & \sum_{\tau=1}^{t} \xi^{d}_{j\tau} \leq \Gamma_{jt}, \\ & 0 \leq \xi^{d}_{j\tau} \leq 1,\ \forall \tau \leq t. \end{aligned} \tag{5.102}$$

$$\begin{aligned} \min \quad & \Gamma_{jt} \cdot \lambda^{d}_{jt} + \sum_{\tau=1}^{t} \mu^{d}_{j\tau t} \\ \text{s.t.:} \quad & \lambda^{d}_{jt} + \mu^{d}_{j\tau t} \geq \hat{d}_{j\tau},\ \forall \tau \leq t \\ & \lambda^{d}_{jt},\ \mu^{d}_{j\tau t} \geq 0,\ \forall \tau \leq t. \end{aligned} \tag{5.103}$$

The dual auxiliary problems are incorporated into the robust optimization model in order to produce a tractable formulation.

Appendix 5.B Adjustable robust optimization transformations

By applying the worst-case realizations of the demand uncertainty in the ARO model we obtain the following non-linear model:

(F7b: `AppAROModel`)

$$\min \sum_{j\in J}\sum_{t\in TP} H'_{jt} + \sum_{j,\ell\in J}\sum_{t\in TP}\sum_{n\in N_t} s_{j\ell}\cdot Z_{j\ell n} + \sum_{t\in TF}\sum_{j\in J}\sum_{n\in N_t} c_j \cdot X^0_{jn}$$

$$+ \sum_{t\in TF}\sum_{j\in J}\max_{\tilde{\mathbf{d}}\in U}\left(\sum_{n\in N_t} c_j \cdot \sum_{r=|TP|+1}^{t-1} X^r_{jn}\cdot(d_{j\tau}+\hat{d}_{j\tau}\cdot\xi_{j\tau})\right)$$

$$+ \sum_{t\in TF}\sum_{j\in J}\max_{\tilde{\mathbf{d}}\in U}\left(H^0_{jt} + \sum_{r=|TP|+1}^{t-1} H^r_{jt}\cdot(d_{jr}+\hat{d}_{jr}\cdot\xi_{jr})\right) \tag{5.104}$$

s.t.:

$$H'_{jt} \geq h^+_{jt}\cdot\left(I^+_{0t} + \sum_{\tau=1}^{t}(\sum_{n\in N_\tau} X'_{jn} - d_{j\tau})\right),\ \forall j\in J,\ t\in TP \tag{5.105}$$

$$H'_{jt} \geq h^-_{jt}\cdot\left(I^-_{0t} + \sum_{\tau=1}^{t}(\sum_{n\in N_\tau} -X'_{jn} + d_{j\tau})\right),\ \forall j\in J,\ t\in TP \tag{5.106}$$

$$0 \geq -H^0_{jt} + h^+_{jt}\cdot\left(I^+_{0t} + \sum_{\tau=|TP|+1}^{t}\sum_{n\in N_\tau} X^0_{jn} + \sum_{\tau\in TP}(\sum_{n\in N_\tau} X'_{jn} - d_{j\tau})\right)$$

$$+ \max_{\tilde{\mathbf{d}}\in U}\left(\sum_{r=|TP|+1}^{t-1}(\sum_{\tau=r+1}^{t}\sum_{n\in N_\tau} X^r_{jn}\cdot h^+_{jt} - H^r_{jt})\cdot(d_{jr}+\hat{d}_{jr}\cdot\xi_{jr})\right.$$

$$\left. - \sum_{r=|TP|+1}^{t} h^+_{jt}\cdot(d_{jr}+\hat{d}_{jr}\cdot\xi_{jr})\right),\ \forall j\in J,\ t\in TF \tag{5.107}$$

$$0 \geq -H^0_{jt} + h^-_{jt}\cdot\left(I^-_{0t} - \sum_{\tau\in TF}\sum_{n\in N_\tau} X^0_{jn} + \sum_{\tau\in TP}(\sum_{n\in N_\tau} -X'_{jn} + d_{j\tau})\right)$$

$$+ \max_{\tilde{\mathbf{d}}\in U}\left(-\sum_{r=|TP|+1}^{t-1}(\sum_{\tau=r+1}^{t}\sum_{n\in N_\tau} X^r_{jn}\cdot h^-_{jt} + H^r_{jt})\cdot(d_{jr}+\hat{d}_{jr}\cdot\xi_{jr})\right.$$

$$\left. + \sum_{r=|TP|+1}^{t} h^-_{jt}\cdot(d_{jr}+\hat{d}_{jr}\cdot\xi_{jr})\right),\ \forall j\in J,\ t\in TF \tag{5.108}$$

$$\sum_{j\in J}\sum_{n\in N_t} p_j\cdot X'_{jn} + \sum_{j,\ell\in J}\sum_{n\in N_t} q_{j\ell}\cdot Z_{j\ell n} \leq cap_t,\ \forall t\in TP \tag{5.109}$$

$$\sum_{j\in J}\sum_{n\in N_t} u_j \cdot X^0_{jn} + \max_{\tilde{\mathbf{d}}\in U}\left(\sum_{j\in J}\sum_{n\in N_t} u_j \cdot \sum_{r=|TP|+1}^{t-1} X^r_{jn}\cdot(d_{jr}+\hat{d}_{jr}\cdot\xi_{j\tau})\right) \le cap_t,\ \forall t\in TF \tag{5.110}$$

$$X'_{jn} \le b_{jt}\cdot Y_{jn},\ \forall j\in J,\ t\in TP,\ n\in N_t \tag{5.111}$$

$$\sum_{j\in J} Y_{jn} = 1,\ \forall t\in TP,\ n\in N \tag{5.112}$$

$$\sum_{\ell\in J} Z_{j\ell n} = Y_{j(n-1)},\ \forall t\in TP,\ j\in J,\ n\in N_t \tag{5.113}$$

$$\sum_{j\in J} Z_{j\ell n} = Y_{\ell n},\ \forall \ell\in J,\ t\in TP,\ n\in N_t \tag{5.114}$$

$$X'_{jn} \ge m_j\cdot\left(Y_{jn}-Y_{j(n-1)}\right),\ \forall j\in J,\ t\in TP,\in N_t \tag{5.115}$$

$$X^0_{jn} + \min_{\tilde{\mathbf{d}}\in U}\left(\sum_{r=|TP|+1}^{t-1} X^r_{jn}\cdot(d_{jr}+\hat{d}_{jr})\right) \ge 0,\ \forall j\in J,\ t\in TF,\ n\in N_t \tag{5.116}$$

$$H^0_{jt} + \min_{\tilde{\mathbf{d}}\in U}\left(\sum_{r=|TP|+1}^{t-1} H^r_{jt}\cdot(d_{jr}+\hat{d}_{jr})\right) \ge 0,\ \forall j\in J,\ t\in TF \tag{5.117}$$

$$Y_{jn}\in\mathbb{B},\ X'_{jn}, H'_{jt},\ Z_{j\ell n}\ge 0,\ \forall j,\ell\in J,\ n\in N,\ t\in T. \tag{5.118}$$

Each inner optimization problem can be transformed in an auxiliary primal and respective dual problem for each $(j,t,\ \forall t\in TF)$:

$$\begin{aligned} \max\quad & \sum_{\tau=1}^{t-1}\sum_{n\in N_t} c_j\cdot X^\tau_{jn}\cdot\hat{d}_{j\tau}\cdot\xi^d_{j\tau} \\ \text{s.t.:}\quad & \sum_{\tau=1}^{t-1}\xi^d_{j\tau}\le\Gamma_{jt-1}, \\ & 0\le\xi^d_{j\tau}\le 1,\ \forall\tau<t. \end{aligned} \tag{5.119}$$

$$\begin{aligned} \min\quad & \Gamma_{jt-1}\cdot\lambda^A_{jt}+\sum_{\tau=1}^{t-1}\mu^A_{j\tau t} \\ \text{s.t.:}\quad & \lambda^A_{jt}+\mu^A_{j\tau t}\ge\sum_{n\in N_t} c_j\cdot X^\tau_{jn}\cdot\hat{d}_{j\tau},\ \forall\tau<t \\ & \lambda^A_{jt},\ \mu^A_{j\tau t}\ge 0,\ \forall\tau<t. \end{aligned} \tag{5.120}$$

$$\begin{aligned}
\max \quad & -\sum_{\tau=1}^{t} \hat{d}_{j\tau} \cdot h^{+}_{jt} \cdot \xi^{d}_{j\tau} + (\sum_{r=1}^{t-1}(\sum_{\tau=r+1}^{t} \sum_{n \in N_\tau} X^{r}_{jn} \cdot h^{+}_{jt} - H^{r}_{jt}) \cdot \hat{d}_{j\tau} \cdot \xi^{d}_{j\tau} \\
\text{s.t.:} \quad & \sum_{\tau=1}^{t} |\xi^{d}_{j\tau}| \leq \Gamma_{jt}, \\
& -1 \leq \xi^{d}_{j\tau} \leq 1, \ \forall \tau \leq t.
\end{aligned} \tag{5.121}$$

$$\begin{aligned}
\max \quad & -\sum_{\tau=1}^{t} \hat{d}_{j\tau} \cdot h^{+}_{jt} \cdot \xi^{d}_{j\tau} + (\sum_{r=1}^{t-1}(\sum_{\tau=r+1}^{t} \sum_{n \in N_\tau} X^{r}_{jn} \cdot h^{+}_{jt} - H^{r}_{jt}) \cdot \hat{d}_{j\tau} \cdot \xi^{d}_{j\tau} \\
\text{s.t.:} \quad & \sum_{\tau=1}^{t} \Xi^{d}_{j\tau} \leq \Gamma_{jt}, \\
& -\Xi^{d}_{j\tau} \leq \xi^{d}_{j\tau} \leq \Xi^{d}_{j\tau}, \ \forall \tau \leq t \\
& \Xi^{d}_{j\tau} \geq 0, \ \forall \tau \leq t \\
& -1 \leq \xi^{d}_{j\tau} \leq 1, \ \forall \tau \leq t.
\end{aligned} \tag{5.122}$$

$$\begin{aligned}
\min \quad & \Gamma_{jt} \cdot \lambda^{B}_{jt} + \sum_{\tau=1}^{t} (\mu^{B1}_{j\tau t} - \mu^{B2}_{j\tau t}) \\
\text{s.t.:} \quad & \mu^{B1}_{j\tau t} + \mu^{B2}_{j\tau t} + \nu^{B1}_{j\tau t} + \nu^{B2}_{j\tau t} = (\sum_{\tau=r+1}^{t} \sum_{n \in N_\tau} X^{r}_{jn} \cdot h^{+}_{jt} - H^{r}_{jt} - h^{+}_{jt}) \cdot \hat{d}_{j\tau}, \ \forall \tau < t \\
& \mu^{B1}_{j\tau t} + \mu^{B2}_{j\tau t} + \nu^{B1}_{j\tau t} + \nu^{B2}_{j\tau t} = -h^{+}_{jt} \cdot \hat{d}_{j\tau}, \ \forall \tau = t \\
& \lambda^{B}_{jt} - \nu^{B1}_{j\tau t} + \nu^{B2}_{j\tau t} \geq 0 \ \forall \tau, \ t \\
& \mu^{B2}_{j\tau t}, \ \nu^{B2}_{j\tau t} \leq 0, \ \forall \tau \leq t \\
& \lambda^{B}_{jt}, \ \mu^{B1}_{j\tau t}, \ \nu^{B1}_{j\tau t} \geq 0, \ \forall \tau \leq t.
\end{aligned} \tag{5.123}$$

$$\begin{aligned}
\max \quad & \sum_{\tau=1}^{t} \hat{d}_{j\tau} \cdot h^{-}_{jt} \cdot \xi^{d}_{j\tau} - \sum_{r=1}^{t-1}(\sum_{\tau=r+1}^{t} \sum_{n \in N_\tau} X^{r}_{jn} \cdot h^{-}_{jt} - H^{r}_{jt}) \cdot \hat{d}_{j\tau} \cdot \xi^{d}_{j\tau} \\
\text{s.t.:} \quad & \sum_{\tau=1}^{t} |\xi^{d}_{j\tau}| \leq \Gamma_{jt}, \\
& -1 \leq \xi^{d}_{j\tau} \leq 1, \ \forall \tau < t.
\end{aligned} \tag{5.124}$$

$$
\begin{aligned}
\max \quad & \sum_{\tau=1}^{t} \hat{d}_{j\tau} \cdot h^{-}_{jt} \cdot \xi^{d}_{j\tau} - \sum_{r=1}^{t-1} \Big(\sum_{\tau=r+1}^{t} \sum_{n \in N_\tau} X^{r}_{jn} \cdot h^{-}_{jt} - H^{r}_{jt} \Big) \cdot \hat{d}_{j\tau} \cdot \xi^{d}_{j\tau} \\
\text{s.t.:} \quad & \sum_{\tau=1}^{t} \Xi^{d}_{j\tau} \le \Gamma_{jt}, \\
& -\Xi^{d}_{j\tau} \le \xi^{d}_{j\tau} \le \Xi^{d}_{j\tau}, \ \forall \tau \le t \\
& \Xi^{d}_{j\tau} \ge 0, \ \forall \tau \le t \\
& -1 \le \xi^{d}_{j\tau} \le 1, \ \forall \tau \le t.
\end{aligned}
\tag{5.125}
$$

$$
\begin{aligned}
\min \quad & \Gamma_{jt} \cdot \lambda^{C}_{jt} + \sum_{\tau=1}^{t} \mu^{C}_{j\tau t} \\
\text{s.t.:} \quad & \mu^{C1}_{j\tau t} + \mu^{C2}_{j\tau t} + \nu^{C1}_{j\tau t} + \nu^{C2}_{j\tau t} = \Big(h^{-}_{jt} - \sum_{\tau=r+1}^{t} \sum_{n \in N_\tau} X^{r}_{jn} \cdot h^{-}_{jt} - H^{r}_{jt} \Big) \cdot \hat{d}_{j\tau}, \ \forall \tau \ < t \\
& \mu^{C1}_{j\tau t} + \mu^{C2}_{j\tau t} + \nu^{C1}_{j\tau t} + \nu^{C2}_{j\tau t} = h^{-}_{jt} \cdot \hat{d}_{j\tau}, \ \forall \tau = t \\
& \lambda^{C}_{jt} - \nu^{C1}_{j\tau t} + \nu^{C2}_{j\tau t} \ge 0 \ \forall \tau, \ t \\
& \mu^{C2}_{j\tau t}, \ \nu^{C2}_{j\tau t} \le 0, \ \forall \tau \le t \\
& \lambda^{C}_{jt}, \ \mu^{C1}_{j\tau t}, \ \nu^{C1}_{j\tau t} \ge 0, \ \forall \tau \le t.
\end{aligned}
\tag{5.126}
$$

$$
\begin{aligned}
\max \quad & \sum_{\tau=1}^{t-1} \sum_{n \in N_t} u_j \cdot X^{\tau}_{jn} \cdot \hat{d}_{j\tau} \cdot \xi^{d}_{j\tau} \\
\text{s.t.:} \quad & \sum_{\tau=1}^{t-1} \xi^{d}_{j\tau} \le \Gamma_{jt-1}, \\
& 0 \le \xi^{d}_{j\tau} \le 1, \ \forall \tau < t.
\end{aligned}
\tag{5.127}
$$

$$
\begin{aligned}
\min \quad & \Gamma_{jt-1} \cdot \lambda^{D}_{jt} + \sum_{\tau=1}^{t-1} \mu^{D}_{j\tau t} \\
\text{s.t.:} \quad & \lambda^{D}_{jt} + \mu^{D}_{j\tau t} \ge \sum_{n \in N_t} u_j \cdot X^{\tau}_{jn} \cdot \hat{d}_{j\tau}, \ \forall \tau < t \\
& \lambda^{D}_{jt}, \ \mu^{D}_{j\tau t} \ge 0, \ \forall \tau < t.
\end{aligned}
\tag{5.128}
$$

$$
\begin{aligned}
\max \quad & \sum_{\tau=1}^{t-1} X^{\tau}_{jn} \cdot \hat{d}_{j\tau} \cdot \xi^{d}_{j\tau} \\
\text{s.t.:} \quad & \sum_{\tau=1}^{t-1} \xi^{d}_{j\tau} \le \Gamma_{jt-1}, \\
& 0 \le \xi^{d}_{j\tau} \le 1, \ \forall \tau < t.
\end{aligned}
\tag{5.129}
$$

$$
\begin{aligned}
\min \quad & \Gamma_{jt-1} \cdot \lambda^{E}_{jn} + \sum_{\tau=1}^{t-1} \mu^{E}_{j\tau n} \\
\text{s.t.:} \quad & \lambda^{E}_{jn} + \mu^{E}_{j\tau n} \ge X^{\tau}_{jn} \cdot \hat{d}_{j\tau}, \ \forall \tau < t, \ n \in N_t \\
& \lambda^{E}_{jt}, \ \mu^{E}_{j\tau t} \ge 0, \ \forall \tau < t.
\end{aligned}
\tag{5.130}
$$

$$\begin{aligned} \max \quad & \sum_{\tau=1}^{t-1} H_{jt}^{\tau} \cdot \hat{d}_{j\tau} \cdot \xi_{j\tau}^{d} \\ \text{s.t.:} \quad & \sum_{\tau=1}^{t-1} \xi_{j\tau}^{d} \leq \Gamma_{jt-1}, \\ & 0 \leq \xi_{j\tau}^{d} \leq 1, \ \forall \tau < t. \end{aligned} \tag{5.131}$$

$$\begin{aligned} \min \quad & \Gamma_{jt-1} \cdot \lambda_{jt}^{F} + \sum_{\tau=1}^{t-1} \mu_{j\tau t}^{F} \\ \text{s.t.:} \quad & \lambda_{jt}^{F} + \mu_{j\tau t}^{F} \geq H_{jt}^{\tau} \cdot \hat{d}_{j\tau}, \ \forall \tau < t \\ & \lambda_{jt}^{F}, \ \mu_{j\tau t}^{F} \geq 0, \ \forall \tau < t. \end{aligned} \tag{5.132}$$

Therefore, the dual auxiliary problems can be incorporated into the ARO model in order to produce a tractable tractable formulation.

CHAPTER 6

Conclusions and future work

This thesis approaches studies the integration of lot-sizing with other planning decisions and the acknowledgement of uncertainty. The work can be divided into three branches. The first is the review of the literature in order to classify and identify state-of-the-art approaches regarding uncertainty and integration in lot-sizing, as well as the respective main gaps. The second is focused on developing relevant integrated lot-sizing models that are able to manage uncertainty and that have not been addressed in the literature. The third is more results-oriented, in which the models and solution approaches developed are assessed in terms of computational complexity, quality of solutions and risk measures. Chapter 2 is dedicated to the first branch of this research, in which we point out the main contributions, research opportunities and gaps in the literature of deterministic, uncertainty and integrated lot-sizing problems. Chapters 3, 4 and 5 are aligned with the second research branch, devoting to new models that integrate lot-sizing with other planning decisions and incorporate critical uncertainty sources. Finally, Chapters 3 and 5 also contribute to the third branch, as underlying works suggest strategies to solve large instances of the problems in a more tractable form. Bellow we first discuss the main contributions of each chapter, then we provide our contribution by answering the research questions raised and finally point directions for further research.

6.1. Contributions

Chapter 3 brings several insights about integration and incorporation of uncertainty focused on food processing industry. The integration of decisions accounted for significant savings: results show that integration of supplier selection, product branding and tactical planning decisions can improve up to 7.4% the solution quality, when compared to decoupled models. Moreover, the incorporation of relevant specific features of food industry, such as shelf-life, customer willingness to pay and critical uncertainty sources helps by delivering good solution quality for the problem. The incorporation of risk measures for the stochastic programming model was also important to stablish a trade-off on expected profit and profits in worst-case scenarios. Finally, the main insight provided by this work shows that decomposition and acceleration schemes are also powerful for complex problems: modern Benders decomposition technique was up to five times more efficient than the monolithic model solved with CPLEX. Generally, integrated and stochastic problems have a natural decomposition structure that can exploited by reformulations and decompositions approaches to make them more tractable. Nevertheless, decomposition methods are not guaranteed to improve the solution efficiency. Sometimes, it is necessary to test several decomposition and acceleration schemes in order to select the best one.

The contributions of Chapter 4 are two-fold. The first is formulating a robust optimization and a two-stage stochastic programming model for the lot-sizing and scheduling problem. The models incorporate uncertainty in demand and assume a fixed production quantity and sequence for the entire planning horizon, allowing only inventory decisions to be adjusted in every time period. To the best of our knowledge, it is the first time that GLSP has been tackled using a robust optimization model. The second contribution is the development of a method to evaluate aspects of the different uncertainty modeling approaches. The simulation experiment allows the trade-off analysis of robust and stochastic models according to different risk preferences and instances characteristics. For example, in many instances the robust optimization model yields almost the same solution quality in terms of expected costs as the stochastic programming model, showing that it is possible to have a tractable model that incorporates demand uncertainty and generates adequate solutions for the problem. Moreover, the simulation showed itself to be an useful tool to fine tune the robust optimization parameters, allowing for a precise measure of risk and performance trade-off. Finally, based on the simulation experiments, a novel flowchart contributed in providing guidelines to select the best modeling approach and uncertainty parameters for different settings.

Chapter 5 presents adaptation and approximate strategies for solving efficiently the lot-sizing and scheduling problem under multistage demand uncertainty. Focusing on solution quality and computational complexity, the proposed heuristic strategies outperformed the standard approaches in terms of efficiency for large instances and long planning horizons: in our experiments, the strategies proposed delivered up to 6.72% better performance and 7.9 times faster results than the deterministic model. We believe that it is the first time that an approximate strategy is used in uncertainty lot-sizing problems and, to the best of our knowledge, adjustable robust optimization had never been used before to address lot-sizing and scheduling problems. Given the absence of standardized evaluation methods, this work also presents a simulation experiment based on Monte Carlo and rolling-horizon schemes in order to systematically compare the models and strategies proposed. This turned out to be a reliable method for evaluating the performance of the models and strategies in multistage uncertainty settings.

6.2. Answering the research questions

Here, we give our contribution in answering the research questions raised. There is not an easy and unique answer for each question raised. Therefore, we provide our answers based on the works already discussed and focused on the integrated problems addressed. We also derive general guidelines that may help the scientific community and decision makers to approach other integrated lot-sizing problems under uncertainty in a more effective and efficient manner.

Research question 1:

What is the most adequate approach to model specific integrated lot-sizing problems *under uncertainty?*

Some aspects can define the adequate modeling approach regarding uncertainty. After identifying the critical uncertainty sources to be incorporated, it is necessary to check if the uncertainties can be defined from previous data or a distribution curve, and whether recursive decisions are required. In case it is not possible to generate credible scenarios through previous data or distribution curves, robust optimization may be the appropriate choice. On the other hand, if recursive decisions are required then static robust optimization should be discarded.

Results suggest that the choice of the modeling approach depends on the problem structure and risk preferences of the decision maker. Despite the known advantages and drawbacks of stochastic and robust approaches, the problems structure and size substantially impacts on the performance of both modeling approaches, as shown in Chapter 4. The length of the planning horizon, size of instances and number of the uncertainty sources may render the stochastic programming approach an intractable option. On the other hand, restrictive production capacity may impact the robust optimization solution quality. If the decision maker is prioritizing conservative decisions, then risk measures (such as conditional value-at-risk applied in Chapter 3) or min-max approaches should be incorporated in the stochastic programming model or the robust optimization model should be used.

Focusing on the integration issue, the decisions to be integrated should be directly related or affect production lot-sizing decisions. Moreover, the integration must have potential to improve the designed objective. The structure of the resulting integrated model should be considered for identifying valid inequalities and for possible reformulations or application of decomposition methods. For instance, Chapter 3 exploits the problem structure to apply convex hull reformulations in order to provide better bounds for the model.

Finally, we believe that it is crucial to evaluate the models proposed and measure their trade-offs for each specific problem and instance characteristic. To that end, a Monte Carlo simulation is suggested. As shown in Chapters 4 and 5, the Monte Carlo simulation experiment allows to measure, in a quantitative manner, several metrics of models in a uncertainty setting, for instance: 1) evaluate and validate the solutions from different models and strategies; 2) tune models uncertainty parameters; 3) evaluate the value of integration, the value of perfect information and the value of uncertainty incorporated; 4) evaluate the cost of assuming a wrong moment information about the uncertainty (e.g., mean, distribution, deviation).

Research question 2:

What are the best strategies to efficiently solve specific integrated lot-sizing problems under uncertainty?

Solving efficiency is achieved by the right combination of modeling approaches and solving techniques. Therefore, we believe that a deep understanding of the problem and integration structure as well as the uncertainty sources to be incorporated and the risks preferences should be the first step in order to select the combination of modelling and solution technique.

Depending on the uncertainty modeling approach, it is possible to maintain the same tractability of the deterministic versions of the problem and provide an adequate solution. Nevertheless, if we are dealing with large problems or intractable uncertainty modeling approaches, such as stochastic programming or adjustable robust optimization, there are specific techniques that can improve the solving efficiency. In case the deterministic version of the problem is already intractable because of the integration, there might be a natural decomposition of the decisions that can be exploited by decomposition methods, such as Benders decomposition, Lagrangian decomposition or cross decomposition. Intractable stochastic programming models can have intractability improved by Sample Average Approximation, scenario reduction techniques, or by decomposition methods previously mentioned. For example, Chapter 3 applies Benders decomposition and Sample Average Approximation approaches to improve the tractability of the stochastic programming model. Splitting uncertainty sets methods can be used for adjustable robust optimization models to deal with adjustable integer variables, still some authors suggest that more research is needed in this field. Therefore, heuristic strategies can be a good alternative, especially for highly intractable problems, such as multistage uncertainty problems. As an example, Chapter 5 shows that by using approximate heuristics we can transform the intractable adjustable robust optimization model to a tractable version that is 7.9 times faster than the deterministic model.

Another way to improve the tractability of lot-sizing models is by means of their reformulation. Lot-sizing problems have alternative formulations, such as the simple plant location model, that provides better relaxations and can be used for lot-sizing problems that do not consider cumulative backlogs. Also, integrated models can be tightened by valid inequalities. For instance, scheduling inequalities can be added to the GLSP model in order to improve its solving efficiency, as applied in Chapter 4. Convex hull reformulations can also provide better relaxation bounds than the standard Big-M formulation, especially when we are dealing with hierarchical decisions, as shown in Chapter 3.

6.3. Future work and research opportunities

The integration of lot-sizing under uncertainty is a broad field and, as mentioned in Chapter 2, there are research opportunities and gaps in the literature that still need attention. Here, we first mention further research related to the works developed and at the end we describe general opportunities and research directions in the field of integrated lot-sizing under uncertainty.

Chapter 3 proposes a two-stage stochastic model that integrates supplier selection with tactical planning decisions. There are opportunities lying on the development of a more complex and detailed model, considering setup decisions and supplier contracts. Additional work can also be performed regarding the application of a tractable robust optimization model to address the problem and its comparison with the stochastic programming model. Decomposition algorithms may also be more explored in order to improve the solving efficiency of the problem. Research on the efficiency of Lagrangian decomposition, cross decomposition or heuristic-generated cuts for the problem seems also promising.

Chapter 4 addresses demand uncertainty in a lot-sizing and scheduling problem. The proposed models can be extended to incorporate system uncertainties, such as uncertain setup or processing times. In terms of solution performance, the study of methods to reduce the conservativeness of robust optimization solutions and improve its solution quality is a possible research direction. Finally, we believe that experiments and methods that compare uncertainty modeling approaches can be improved and standardized. These methods should allow to systematically compare robust optimization and stochastic programming models in a more efficient manner and grasp their best characteristics for different problems and circumstances.

Chapter 5 presents strategies for solving the GLSP under multistage demand uncertainty. Even though these strategies showed to be efficient, they can still be improved. Opportunities lie on the study of machine learning to create precise approximations and rules in order to improve solutions in the rolling-horizon setting. Moreover, it would be valuable to compare these strategies with meta-heuristics and other heuristics (e.g., hierarchical approaches, rolling-horizon heuristics). The study of these strategies in other lot-sizing settings (e.g., multi-level lot-sizing, lot-sizing with parallel machines) is also a possible research direction.

Besides the research opportunities related to the work developed, we describe the gaps identified in the literature review that have not been addressed in this thesis. Firstly, there is still a lack of works that consider system uncertainty and complex lot-sizing models. Models that combine several uncertainty sources or take into account complex lot-sizing problems can bring superior solutions for practical problems. Nevertheless, we agree that these models would be highly intractable and specific attention on solving efficiency would be required.

There are few studies that have addressed the integration, in a uncertainty setting, of lot-sizing with classical problems (e.g., vehicle routing problem, capacity/facility planning, procurement planning, cutting problems, make-or-buy decisions). Research in this direction may have high practical application. A research opportunity also lies on the study and comparison of risk-averse strategies (robust optimization models and the incorporation of risk measures in stochastic models) in integrated lot-sizing models. Works that analyze, for different problems and uncertainty sources, the trade-offs in terms of solution quality and risk reduction could bring significant contributions to the scientific community.

Moreover, there are only few studies that assess the cost of assuming a wrong information about the uncertainty source or evaluate the performance of models when the uncertainty moments are partially known. Studies in these directions can provide guidelines to best deal with uncertainty in these cases.

Finally, one of the major issues of uncertainty problems is the intractability of the underlying modeling approaches. An emerging research stream is the study of efficient solution methods for complex integrated lot-sizing problems under uncertainty. Tractability in uncertainty models would allow to incorporate more realistic features and expand the application to practical problems. We suggest two research streams to contribute in this direction. The first is the study of tractable robust optimization models for complex problems. The development of methods to reduce conservativeness would be extremely valuable in improving the average solution quality of robust optimization models. The second stream is on the study of methods (e.g., decomposition, approximation, heuristics) in order to bring tractability for specific modeling approaches, especially for multistage stochastic programming and intractable robust optimization approaches.